Chapter 22 – Inference for a Population Proportion

22.1 (a) The population is surgical patients. p is the proportion of all surgical patients who will test positive for *Staphylococcus aureus*. **(b)** $\hat{p} = \frac{1251}{6771} = 0.185$ or 18.5%.

22.2 (a) p = 59969/461299 = 0.13. **(b)** Since n = 689 is large, the distribution of the sample proportion under repeated sampling is approximately Normal, with mean p = 0.13 and standard deviation $\sqrt{\frac{p(1-p)}{n}} = 0.0128$. About 95% of sample proportions will be within 2 standard deviations of the true value of p, or between $0.13 - 2(0.0128) = 0.1044$ and $0.13 + 2(0.0128) = 0.1556$. **(c)** Yes. Since cell phone numbers were not included, this survey is not representative of the population of all adults. For instance, younger adults are likely underrepresented, since this group is less likely to use landline phones.

22.3 (a) Since n is large, the distribution of $\hat{p}$ is approximately Normal, with mean $p = 0.3$ and standard deviation $\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.3(0.7)}{1500}} = 0.0118$. **(b)** If the sample size were 6000, the standard deviation of the distribution becomes $\sqrt{\frac{0.3(0.7)}{6000}} = 0.0059$.

22.4 There were only five or six successes in the sample (because 5/2673 and 6/2673 both round to 0.2%).

22.5 (a) The survey excludes residents of the northern territories, as well as those who have no phones or have only cell phone service. **(b)** $\hat{p} = \frac{1288}{1505} = 0.8558$, so $SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ = 0.009055, and the 95% confidence interval is 0.8558 ± (1.96)(0.009055) = 0.8381 to 0.8735, or 83.8% to 87.4%.

22.6 STATE: What proportion of weightlifting injuries in the 8 to 30 age group are accidental? PLAN: We will construct a 90% confidence interval for p. SOLVE: We are told that the sample is random and will assume that the sample is close to an SRS. Since both the number of successes (1552) and failures (4111 − 1552 = 2559) are much greater than 15, we may assume that the sampling distribution of $\hat{p}$ is approximately Normal. Here, $\hat{p} = \frac{1552}{4111}$ = 0.3775 and $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{4111}}$ = 0.0076. A 90% confidence interval for p is given by 0.3775 $\pm$ 1.645(0.0076) = 0.365 to 0.390. CONCLUDE: With 90% confidence, the proportion of all weightlifting injuries in this age group that are classified as accidental is between 0.365 and 0.390.

22.7 (a) We have $\hat{p} = \frac{52}{356} = 0.146$, so the margin of error is $1.96\sqrt{\frac{0.146(1-0.146)}{356}} = 0.0367$. **(b)** To get $m = 0.03$ with $p^* = 0.146$, we need a sample size of $n = \left(\frac{1.96}{0.03}\right)^2 (0.146)(1-0.146) = 532.2$. So we need at least 533 visitors over age 65.

22.8 (a) $n = \left(\frac{z^*}{m}\right)^2 p^*(1-p^*) = \left(\frac{1.645}{0.04}\right)^2 (0.75)(1-0.75)$ = 317.1, so use n = 318. **(b)** With no assumptions about p, we would need $n = \left(\frac{1.645}{0.04}\right)^2 (0.50)(1-0.50)$ = 422.82; so 423.

22.9 STATE: We wonder if the proportion of student judges who correctly identify the dog-owner pairs is the same as the proportion who do not correctly identify them for both cases of the mouth region or eye region being blacked out. PLAN: Take p_1 to be the proportion of undergraduates who would correctly match the sheet when the mouth region is blacked out. We want to test the hypotheses $H_0: p_1 = 0.5$ $H_a: p_1 > 0.5$. Let p_2 be the proportion for the case when the eye region is blacked out. We also want to test $H_0: p_2 = 0.5$ against $H_a: p_2 > 0.5$. SOLVE: For the case where the mouth region is blacked out: We expect 51(0.5) = 25.5 successes and 25.5 failures. Both are larger than 10 so, assuming this sample can be treated like an SRS, the conditions are met. $\hat{p} = \frac{37}{51} = 0.725$, $z = \frac{0.725 - 0.5}{\sqrt{\frac{0.5(0.5)}{51}}} = 3.21$, and the P-value is 0.0007. For the case where the eye region is blacked out: we expect 60(0.5) = 30 successes and 30 failures. Assuming we have an SRS, the conditions are met. $\hat{p} = \frac{30}{60} = 0.5$, $z = \frac{0.5 - 0.5}{\sqrt{\frac{0.5(0.5)}{60}}} = 0$, and the P-value is 0.5. CONCLUDE: When the mouth region is blacked out, there is strong evidence that the proportion of students who can correctly match the dog and owner pairs is greater than 0.5. When the eye region is blacked out, there is not evidence that more than half of students can correctly identify the dog and owner pairs. This study suggests the eyes are the primary feature that enables one to identify the dog-owner pairs.

22.10 STATE: If the students were choosing randomly, each of the five pairs of socks would be equally likely to be chosen. Thus, we wonder if the proportion of times the subject choosing the center sock is more than 1/5 = 0.2. PLAN: Let p be the proportion of times the center sock is chosen. We test $H_0: p = 0.2$ versus $H_a: p > 0.20$. SOLVE: Since the sample consists of 100 trials, we expect $100(0.2) = 20$ successes and $100(1 - 0.2) = 80$ failures. Both of these are at least 10, so the sample is large enough to use the z test. Assume the sample can be thought of as an SRS. Here $\hat{p} = \frac{34}{100} = 0.34$, $z = \frac{0.34 - 0.2}{\sqrt{\frac{0.2(0.8)}{100}}} = 3.5$, and $P \approx 0.0002$. CONCLUDE: There is strong evidence that the proportion of times the center sock is chosen is more than 0.2.

22.11 (a) The number of trials is not large enough to apply the z test for a proportion. Here, the expected number of successes (heads) and the expected number of failures (tails) are both 5; these should be 10 or more. **(b)** As long as the sample can be viewed as an SRS, a z test for a proportion can be used. **(c)** Under the null hypothesis, we expect only 200(0.01) = 2 failures. We should have at least 10 expected failures and at least 10 expected successes.

22.12 (a) Among the 14 observations, we have 11 successes and 3 failures. The number of successes and failures should both be at least 15 for the large-sample confidence interval to be valid. **(b)** We add 4 observations: 2 successes and 2 failures. We now have 18 observations: 13 successes and 5 failures. Now $\tilde{p} = \frac{11+2}{14+4} = \frac{13}{18}$ = 0.7222. **(c)** Using the plus four method, $SE = \sqrt{\frac{0.7222(1-0.7222)}{18}}$ = 0.1056. A 90% confidence interval for p is 0.7222 $\pm$ 1.645(0.1056) = 0.5485 to 0.8959. The confidence interval is quite wide (even with only 90% confidence used) because the sample size is so small.

22.13 (a) We are told we have a random sample. There are 185 successes and $925 - 185 = 740$ failures; both are at least 15. $\hat{p} = \frac{185}{925} = 0.2$. A large-sample 95% confidence interval for p, the proportion who consider themselves "gamers," is 0.2 $\pm$ 1.96 $\sqrt{\frac{0.2(0.8)}{925}} = 0.174$ to 0.226, or 17.4% to 22.6%. **(b)** The conditions are met for the plus four interval because the sample size is at least 10. $\tilde{p} = \frac{185\,+\,2}{925\,+\,4} = 0.201$. A plus four 95% confidence interval is 0.201 $\pm$ 1.96 $\sqrt{\frac{0.201(1\,-\,0.201)}{929}} = 0.175$ to 0.227, or 17.5% to 22.7%. If the two intervals are rounded to the nearest tenth of a percent, the plus four interval is one-tenth greater than the large sample interval. The shift is very small because the sample is so large.

22.14 (a) The sample proportion is $\hat{p}$ = $\frac{20}{20}$ = 1, so $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ = 0. The margin of error would therefore be 0 (regardless of the confidence level), so large-sample methods give the useless interval 1 to 1. **(b)** The plus four estimate is $\tilde{p} = \frac{20\,+\,2}{20\,+\,4}$ = 0.9167, and $SE_{\tilde{p}} = \sqrt{\frac{\tilde{p}(1-\tilde{p})}{24}}$ = 0.0564. A 95% confidence interval for p is then 0.9167 $\pm$ 1.96(0.0564) = 0.8062 to 1.0272. Because proportions can't exceed 1, we say that a 95% confidence interval for p is 0.8062 to 1.

22.15 (b) 0.6. The sampling distribution of $\hat{p}$ has mean p = 0.6.

22.16 (b) 0.045. The standard deviation of $\hat{p}$ is $\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(1-0.6)}{117}}$ = 0.0453.

22.17 (b) 0.611 ± 0.025. The 90% confidence interval is $0.611 \pm 1.645\sqrt{\frac{0.611(1-0.611)}{1009}}$.

22.18 (b) would have a larger margin of error than the 90% confidence interval. Less confidence means a smaller margin of error.

22.19 (c) n = 16590. $n = \left(\frac{2.576}{0.01}\right)^2(0.5)(0.5) = 16589.44$, so we would need at least 16590.

22.20 (b) 0.530 ± 0.098. With $\hat{p} = \frac{53}{100}$ = 0.53, the margin of error is $1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ = $1.96\sqrt{\frac{0.53(1-0.53)}{100}}$ = 0.098.

22.21 (a) is in addition to the margin of error found in Exercise 22.20. Sources of bias are not accounted for in a margin of error.

22.22 (a) H_0: p = 0.5, H_a: p > 0.5. The alternative hypothesis expresses the idea "more will finish the maze faster with the loud noise."

22.23 (c) significant at both α = 0.05 and α = 0.01. The P-value is 0.0057.

22.24 (a) the poll used a method that gets an answer within 4% of the truth about the population 95% of the time. This is what 95% confidence means.

22.25 (a) The survey excludes those who have no phones or have only cell phone service. **(b)** Note that we have 848 Yes answers and 162 No answers; both of these are at least 15. With the sample proportion $\hat{p} = \frac{848}{1010}$ = 0.8396, the large-sample 95% confidence interval is $0.8396 \pm 1.96\sqrt{\frac{0.8396(1-0.8396)}{1010}}$ = 0.8170 to 0.8622.

22.26 We have 19 successes and 172 − 19 = 153 failures. Both are at least 15, so the conditions for large-sample confidence interval use are met. We estimate that $\hat{p} = \frac{19}{172}$ = 0.1105, $SE = \sqrt{\frac{0.1105(1-0.1105)}{172}}$ = 0.02391, the margin of error is 1.96 SE = 0.04686, and the 95% confidence interval is 0.0636 to 0.1574.

22.27 (a) With $\hat{p} = \frac{848}{1010}$ = 0.8396, SE = 0.01155, so the margin of error is 1.96 SE = 0.02263 = 2.26%. **(b)** If instead $\hat{p}$ = 0.5, then SE = 0.01573 and the margin of error for 95% confidence would be 1.96 SE = 0.03084 = 3.08%. **(c)** For samples of about this size, the margin of error is no more than about $\pm$3%, no matter what $\hat{p}$ is.

22.28 (a) We can perform a hypothesis test, since $689(0.13) = 89.57 \geq 10$ and $689(1-0.13) = 599.43 \geq 10$. We are also told to assume we have an SRS. The hypotheses are $H_0: p = 0.13$ and $H_a: p \neq 0.13$. We use the two-tailed alternate, because either too high or too low a proportion of adults over 65 in the sample would mean it is not representative in terms of age of responders. The sample proportion of adults over age 65 is $\hat{p} = \frac{253}{689} = 0.367$. The test statistic is $z = \frac{0.367 - 0.13}{\sqrt{\frac{0.13(1-0.13)}{689}}} = 18.5$, with a P-value of approximately 0. This is overwhelming evidence that the sample does not represent the population of Greenville in terms of age. **(b)** The direction of bias would be such that 24.9% underestimates the actual percent of Greenville adult residents who use the trail, since younger adults who use the trail more often were underestimated in the sample.

22.29 (a) The survey excludes residents of Alaska and Hawaii, and those who do not have cell phone service. **(b)** We have 422 successes and 2063 failures (both at least 15), so the sample is large enough to use the large-sample confidence interval. We have $\hat{p} = \frac{422}{2485} = 0.1698$, and $SE = \sqrt{\frac{0.1698(1-0.1698)}{2485}} = 0.0075$. For 90% confidence, the margin of error is 1.645 SE = 0.0124 and the confidence interval is 0.1574 to 0.1822, or 15.7% to 18.2%. **(c)** Perhaps people who use cell phones to search for information online are younger and more interested in sexually related topics.

22.30 (a) We can construct a large-sample interval, since there are 229 successes and 400 − 229 = 171 failures. We will assume these 400 trials can be thought of as an SRS of all such crosswalk instances. Note $\hat{p} = \frac{229}{400} = 0.5725$. A 95% confidence interval is $0.5725 \pm 1.96\sqrt{\frac{0.5725(1-0.5725)}{400}} = 0.5240$ to 0.6210. **(b)** We have more than 15 successes and failures, and we will assume this can be thought of as an SRS. $\hat{p} = \frac{277}{400} = 0.6925$. A 95% confidence interval is $0.6925 \pm 1.96\sqrt{\frac{0.6925(1-0.6925)}{400}} =$ 0.6473 to 0.7377. **(c)** The confidence interval in part (b) being entirely above the interval in part (a) suggests that drivers are more likely to stop for a pedestrian who is smiling than for one with a neutral expression.

22.31 (a) The large-sample interval is safe because we have 880 trials, with 171 successes and 880 − 171 = 709 failures. For the large-sample interval, $\hat{p} = \frac{171}{880} =$ 0.1943, $SE = \sqrt{\frac{(0.1943)(1-0.1943)}{880}} = 0.01334$, the margin of error is 1.96 SE = 0.02614, and the 95% confidence interval is 0.1682 to 0.2204. **(b)** It is likely that more than 171 respondents have run red lights. We would not expect very many people to claim that they have run red lights when they have not, but some people will deny running red lights when they have.

22.32 (a) Answers will vary, but staff may have visited moderate- to high-use areas in order to get a large number of respondents. **(b)** There are 694 successes (day users) and 325 failures (overnight users). Both are larger than 15, so we can use the large-sample interval. $\hat{p} = \frac{694}{1019} = 0.681$, and a 90% confidence interval is $0.681 \pm 1.645\sqrt{\frac{0.681(1-0.681)}{1019}} = 0.657$ to 0.705. **(c)** Yes. Since the response rate was very similar for the two groups, it seems like nonresponse is not related to the particular group and there is less concern for nonresponse bias. **(d)** It is more appropriate to refer to the interval as a confidence interval for the proportion of day users on the most popular trails in the park, because only users on these trails were sampled. Users of less popular trails may vary from this group with regard to whether they are day hikers or overnight hikers, in which case, our results would be biased for the proportion of day users on all trails.

22.33 (a) The margin of error will slightly change, because the sample proportion of returns claiming itemized deductions will change from state to state. These are likely similar from one state to another, so the margin of error will be similar for the states. **(b)** Yes, it will change because the sample size used for each margin of error will be very different. The sample size for Wyoming will be 500,000(0.01) = 5000, but the sample size for California will be 30,000,000(0.01) = 300,000.

22.34 (a) The margins of error are $1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{100}}$ (shown in the table below).

	$\hat{p}$	0.1	0.3	0.5	0.7	0.9
(a)	m.e.	0.0588	0.0898	0.0980	0.0898	0.0588
(b)	m.e.	0.0263	0.0402	0.0438	0.0402	0.0263

(b) With n = 500, the margins of error are $1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{500}}$. The new margins of error are less than half their former size.

22.35 PLAN: We construct a 99% confidence interval for the proportion of all 17-year-old students in 2012 who had at least one parent graduate from college. SOLVE: We are told to treat this sample as an SRS of 17-year-olds still in school. The sample size is very large, with n = 9000. We are told that $\hat{p}$ = 0.51, so we have 9000(0.51) = 4590 students with a parent who graduated from college and 4410 who did not have a parent who graduated from college. The number of successes and failures is very large (more than the required 15), so the large-sample confidence interval is appropriate. We have $SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{9000}}$ = 0.0053, so the margin of error for 99% confidence is 2.576(0.0053) = 0.0137, and a 99% confidence interval for the proportion is 0.51 ± 0.0137, or 0.4963 to 0.5237, or 49.6% to 52.4%. CONCLUDE: With 99% confidence, the proportion of 17-year-old students still in school with at least one parent who graduated from college is between about 0.496 and 0.524. **(b)** Answers will vary, but perhaps parents who are college graduates are more likely to have children finish school. If so, then the proportion for the entire population is likely to be lower.

22.36 PLAN: With p representing the proportion of songs downloaded by Lucretia, we test $H_0 : p = 0.5$ versus $H_a : p \neq 0.5$. The test is two-sided because we wonder if the proportion downloaded by Lucretia differs from that downloaded by Stan, which would mean that p differs from 0.5. SOLVE: We assume that the 40 songs sampled are an SRS. With 40 songs sampled, we expect 40(0.5) = 20 successes and 40(0.5) = 20 failures (which are both at least 10), so conditions for use of the large-sample z test are satisfied. We have $\hat{p} = \frac{27}{40} = 0.675$, so $z = \frac{0.675 - 0.5}{\sqrt{\frac{0.5(0.5)}{40}}} = 2.21$ and the P-value is 0.0271. CONCLUDE: There is strong evidence that the proportion of songs downloaded by Lucretia differs from 0.5. **(b)** The conditions for a large-sample confidence interval are not met, because only 13 failures are observed, which is less than 15.

22.37 (a) PLAN: Let p represent the proportion of adults who read a book in the last 12 months. We want to estimate p with a 95% confidence interval. SOLVE: Assume the sample can be thought of as an SRS. There are 1125 successes (read a book in the last 12 months) and 1520 − 1125 = 395 failures (did not read a book in the last 12 months). The conditions are met to create a large-sample confidence interval. $\hat{p} = \frac{1125}{1520} = 0.74$. A 95% confidence interval is $0.74 \pm 1.96\sqrt{\frac{0.74(0.26)}{1520}} = 0.718$ to 0.762. CONCLUDE: We are 95% confident that the percent of all adults who have read a book in either print or digital format in the last 12 months is between 71.8% and 76.2%. **(b)** PLAN: Let p represent the proportion who read only digital books out of all adults who read a book in the last 12 months. We want to estimate p with a 95% confidence interval. SOLVE: Assume we have an SRS. There are 91 successes (read only digital books) and 1125 − 91 = 1034 failures. The conditions are met for a large-sample confidence interval since both are at least 15. $\hat{p} = \frac{91}{1125} = 0.081$. A 95% confidence interval for the proportion that have read digital books exclusively is $0.081 \pm 1.96\sqrt{\frac{0.081(1-0.081)}{1125}} = 0.0651$ to 0.0969. CONCLUDE: We are 95% confident that the percent of all adult book readers who exclusively read digital books is between 6.51% and 9.69%.

22.38 (a) PLAN: Let p denote the proportion of students who would score a 5 when taught by the teacher using cash incentives. We want to test the hypotheses $H_0: p = 0.15$ and $H_a: p > 0.15$. SOLVE: $\hat{p} = \frac{15}{61} = 0.2459$. If we use the large-sample test, $z = \frac{0.2459 - 0.15}{\sqrt{\frac{0.15(1-0.15)}{61}}} = 2.1$ and the P-value is $P = 0.0179$. CONCLUDE: Assuming the test is appropriate, there is evidence that the proportion who would score a 5 when taught by the teacher using cash incentives is higher than 0.15. Here, we expect 0.15(61) = 9.15 successes; because this is less than 10, there is a possible problem with the use of the z test in this case. Whether we can view this particular class as a simple random sample is questionable. **(b)** Answers will vary. This was not a designed, randomized experiment, so we cannot say the cash incentive caused the increase in 5's.

22.39 (a) PLAN: We have $H_0: p = 0.5$ and $H_a: p \neq 0.5$. The alternate is two-sided, because we want to know if subjects are not equally likely to choose either of the two positions. SOLVE: We assume we have an SRS from the population. With 32 subjects, we expect 16 successes (people who would pick the first wine) and 16 failures (people who would pick the second wine). $\hat{p} = \frac{22}{32} = 0.6875$ and $z = \frac{0.6875-0.50}{\sqrt{0.50(0.50)/32}} = 2.12$, with P-value 2(0.0170) = 0.0340. CONCLUDE: We have strong evidence that people are not equally likely to choose either of two options (of identical wine). It appears they are more likely to select the first presented wine as their preference. **(b)** People who would go out of their way to participate in such a study are presumed to represent the population of all wine drinkers (or adults). The assumption that we have a simple random sample may not be reasonable.

22.40 PLAN: Let p denote the proportion of orders that are filled accurately. We want to construct a 95% confidence interval for p. SOLVE: Assume the orders can be thought of as an SRS of all ethnic fast food. There are 457 − 42 = 415 successes and 42 failures. Both are larger than 15, so a large-sample confidence interval can be constructed. $\hat{p} = \frac{415}{457} = 0.908$. A 95% confidence interval is $0.908 \pm 1.96\sqrt{\frac{0.908(1-0.908)}{457}} = 0.8815$ to 0.9345. CONCLUDE: We are 95% confident that between 88.15% and 93.45% of orders are filled accurately in the ethnic fast food category.

22.41 PLAN: We obtain the sample size required to estimate the proportion of wine tasters who select the first choice to within ± 0.05 with 95% confidence. SOLVE: We guess that the unknown value of p is 0.6875, as computed in Exercise 22.39. $n = \left(\frac{z^*}{m}\right)^2 p^*(1-p^*) = \left(\frac{1.96}{0.05}\right)^2 (0.6875)(1-0.6875)$ = 330.14, so take n = 331. CONCLUDE: We require a sample of at least 331 wine tasters in order to estimate the proportion of tasters who would choose the first option to within 0.05 with 95% confidence.

22.42 (a) We have 22 people who preferred the first presented wine and 10 who preferred the second. Because 10 is less than 15, the conditions for the large-sample confidence interval are not met. **(b)** PLAN: We will give a 90% confidence interval for p, which is the proportion of subjects who would select the first wine choice presented. SOLVE: The sample size is 32, which is more than 10, so conditions for the plus four interval are met. If we use the plus four method: $\tilde{p} = \frac{22+2}{32+4}$ = 0.6667, $SE = \sqrt{\frac{(0.6667)(1-0.6667)}{36}} = 0.0786$, and the 90% confidence interval is 0.5374 and 0.7960, or 53.74% and 79.60%. CONCLUDE: We are 90% confident that the proportion of subjects who would select the first wine is between about 54% and 80%.

22.43 PLAN: We will give a 90% confidence interval for the proportion of all *Krameria cytisoides* shrubs that will resprout after a fire. SOLVE: We assume that the 12 shrubs in the sample can be treated as an SRS. Because the number of resprouting shrubs is just 5, the conditions for a large-sample interval are not met. Using the plus four method: $\tilde{p} = \frac{5+2}{12+4} = 0.4375$, $SE = 0.1240$, the margin of error is $1.645\ SE = 0.2040$, and the 90% confidence interval is 0.2335 to 0.6415. CONCLUDE: We are 90% confident that the proportion of *Krameria cytisoides* shrubs that will resprout after a fire is between about 0.23 and 0.64.

22.44 (a) Provided we have an SRS, the conditions are met, since the total number of observations is at least 10 and the confidence level is greater than 90%. $\tilde{p} = \frac{49+2}{61+4} = 0.785$. The plus four 99% confidence interval is $0.785 \pm 2.576\sqrt{\frac{0.785(1-0.785)}{61+4}} = 0.654$ to 0.916. **(b)** As a percent, the interval is from 65.4% to 91.6%. This interval is shifted slightly—less than the interval in Example 22.7 by about 1.8%.