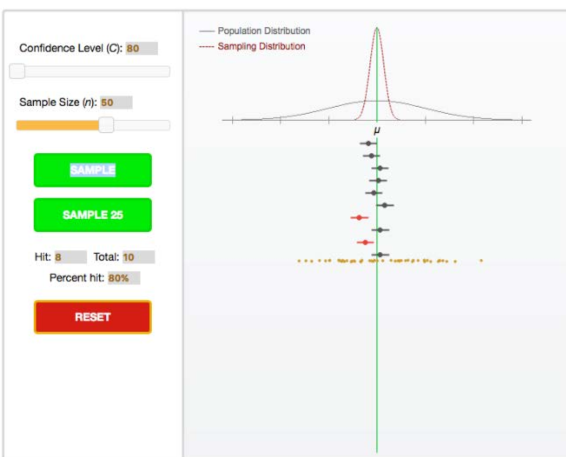
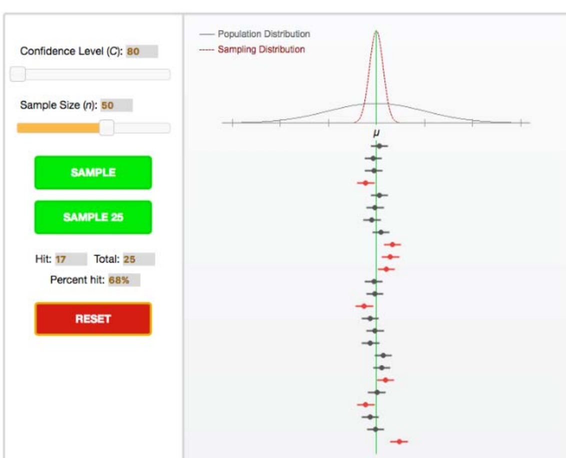
Chapter 16 – Confidence Intervals: The Basics

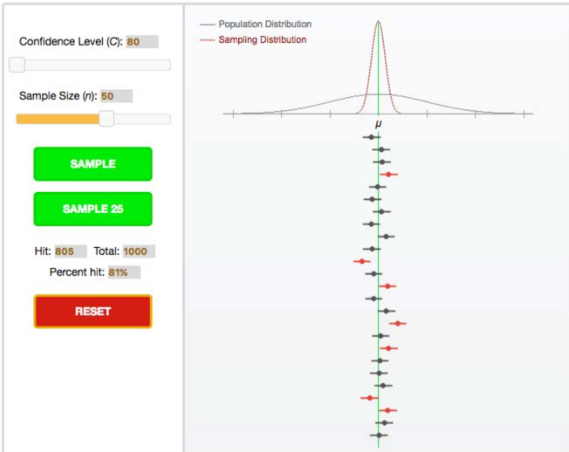
16.1 (a) The sampling distribution of $\bar{x}$ has mean μ (unknown) and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{110}{\sqrt{136,900}} = 0.2973$. **(b)** According to this rule, 95% of all values of $\bar{x}$ fall within 2 standard deviations of the mean μ. That is, within 2(0.2973) = 0.5946 point. **(c)** 282 $\pm$ 0.5946, or between 281.4054 and 282.5946 points.

16.2 (a) The sampling distribution of $\bar{x}$ has mean μ (unknown) and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{400}} = 2$, so a 95% confidence interval is 9 ± 2(2) = 9 ± 4, or between 5 and 13 points. **(b)** The entire interval in part (a) is larger than zero, so we believe that the mean change in score in the population of all high school seniors is greater than zero.

16.3 Answers will vary. Sample output screens are provided for 10, 25, and 1000 SRS's, corresponding to part **(a)**, part **(b)**, and part **(c)**, respectively. In 99.7% of all repetitions of part (a), students should see between 5 and 10 hits (that is, at least 5 of the 10 SRS's capture the true mean μ). Out of 1000 80% confidence intervals, nearly all students will observe between 76% and 84% capturing the mean. This result had 81% of the 1000 intervals containing the mean.







16.4 (a) 26% ± 5%, or 21% to 31%. **(b)** 95% confidence means that this interval was produced using a process for which, in the long run, 95% of all samples of the same size give an interval that contains the actual percent of coffee drinkers in the population (U.S. adults?) who say they are addicted to coffee.

16.5 Search Table A for 0.1250 (half of the 25% that is not included in the middle, shaded area corresponding to 75% confidence). This area corresponds to $-z^* = -1.15$, or $z^* = 1.15$.

16.6 STATE: What is the true melting point of this copper sample? PLAN: We will estimate the true melting point, μ (the mean of all measurements of its melting point), by giving a 90% confidence interval. SOLVE: The statement of the problem in the text suggests that the conditions for inference should be satisfied. The mean of the sample is $\bar{x} = 1084.80$°C. For 90% confidence, the critical value is $z^* = 1.645$. A 90% confidence interval for μ is $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 1084.80 \pm 1.645 \frac{0.25}{\sqrt{6}} = 1084.80 \pm 0.1679 = 1084.6321$ to 1084.9679°C. CONCLUDE: We are 90% confident that the copper sample's true melting point is between 1084.6321 and 1084.9679°C.

16.7 (a) There are no apparent deviations from Normality in the provided stemplot.

Stem and Leaf

Stem	Leaf	Count
13	6	1
13	02	2
12	677888	6
12	003344	6
11	55688999	8
11	0000111122223334444	19
10	555666777789	12
10	0022333344	10
9	6778	4
9	0133	4
8	69	2

(b) STATE: What is the mean IQ μ of all seventh-grade girls in this school district? PLAN: We will estimate μ by giving a 99% confidence interval. SOLVE: The problem states that these girls are an SRS of the population, which is very large. In part (a), we saw that the scores are consistent with having come from a Normal population, so conditions for inference are met. With $\bar{x}$ = 110.73 IQ points and z^* = 2.576, our 99% confidence interval for μ is given by $110.73 \pm 2.576\frac{11}{\sqrt{74}} = 110.73 \pm 3.29 = 107.44$ to 114.02 points.
CONCLUDE: We are 99% confident that the mean IQ of seventh-grade girls in this district is between 107.44 and 114.02 points.

16.8 (a) The three confidence intervals are given in the table provided. In all three cases, $\bar{x}$ = 26.8 and $\frac{\sigma}{\sqrt{n}}$ = 0.2933, so the confidence interval is computed as $26.8 \pm z^*(0.2933)$, where z^* changes with the confidence level.

Confidence Level	z^*	Margin of Error	Interval
90%	1.645	0.4825	26.32 to 27.28
95%	1.960	0.5749	26.23 to 27.37
99%	2.576	0.7555	26.04 to 27.56

(b) The margins of error in part (a) increase as the confidence level increases.

16.9 (a) and **(b)** With z^* = 1.96 and σ = 7.5, the margin of error is $z^*\frac{\sigma}{\sqrt{n}} = \frac{14.7}{\sqrt{n}}$. The margins of error are given in the table.

n	Margin of Error
100	1.4700
400	0.7350
1600	0.3675

(c) Margin of error decreases as n increases. (Specifically, every time the sample size n is quadrupled, the margin of error is halved.)

16.10 (a) With $\bar{x}$ = 9 and z^* = 1.645, our 90% confidence interval for μ is given by $9 \pm 1.645\frac{40}{\sqrt{400}} = 9 \pm 3.29 = 5.71$ to 12.29 points. **(b)** The margin of error is $1.645\frac{40}{\sqrt{400}} = 3.29$ points, which is smaller than the margin of error of 4 that we found in Exercise 16.2 (using z^* = 1.96, this would have been 3.92). **(c)** $1.96\frac{40}{\sqrt{100}} = 7.84$. **(d)** Decreasing the sample size increases the margin of error, provided the confidence level and population standard deviation remain the same.

16.11 (c) z^* = 3.291. Using Table A, search for 0.9995.

16.12 (a) 3.414 ± 0.00113. The margin of error is $z^* \frac{\sigma}{\sqrt{n}} = 1.96 \frac{0.001}{\sqrt{3}}$ = 1.96(0.000577) = 0.00113 gram.

16.13 (b) greater than the margin of error for 95% confidence. As the confidence level increases, z* increases. This makes the margin of error larger.

16.14 (c) 4.1602 ± 0.00091. The margin of error is now $z^* \frac{\sigma}{\sqrt{n}} = 2.576 \frac{0.001}{\sqrt{8}}$ = 0.00091 gram.

16.15 (b) 2.2. The standard deviation of $\bar{x}$ is $\frac{\sigma}{\sqrt{n}} = \frac{110}{\sqrt{2500}} = 2.2$ points.

16.16 (b) 285 ± 4.31. The margin of error is $z^* \frac{\sigma}{\sqrt{n}} = 1.96 \frac{110}{\sqrt{2500}} = 1.96(2.2) = 4.31$, so the confidence interval is 285 ± 4.31. Note that the point estimate alone is enough to determine the solution.

16.17 (a) This 90% confidence interval would have a smaller margin of error than the 95% confidence interval. As the confidence level decreases, z* decreases. This makes the margin of error smaller.

16.18 (b) larger. The smaller the sample size, the larger the margin of error, provided that the confidence level and population standard deviation remain the same.

16.19 (a) We use $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 13.7 \pm 2.576 \frac{7.4}{\sqrt{463}} = 13.7 \pm 0.886 = 12.814$ to 14.586 hours. **(b)** The 463 students in this class must be a random sample of all of the first-year students at this university to satisfy conditions for inference.

16.20 The margin of error for 90% confidence is $1.645 \frac{2.5}{\sqrt{200}} = 0.2908$ kg/m^2, so the interval is 2.35 $\pm$ 0.2908 = 2.0592 to 2.6408 kg/m^2.

16.21 The margin of error is now $2.576 \frac{7.4}{\sqrt{464}}$ = 0.885, so the extra observation has minimal impact on the margin of error (the sample was large to begin with). If $\bar{x}$ = 35.2, then the 99% confidence interval for average amount of time spent studying becomes 35.2 $\pm$ 0.885 $=$ 34.315 to 36.085 hours. The outlier had a huge impact on $\bar{x}$, which shifts the interval a lot.

16.22 The student is incorrect. A 95% confidence interval does not contain 95% of population values. Instead, all we can say is that if we repeatedly sampled the same number of women (with each sample determining a 95% confidence interval for the average perceived ideal weight), then, in the long run, 95% of these confidence intervals would capture the true, unknown average ideal weight as perceived by all American women.

16.23 This student is also confused. If we repeated the sample over and over, 95% of all future sample means would be within 1.96 standard deviations of μ (that is, within $1.96\frac{\sigma}{\sqrt{n}}$) of the true, unknown value of μ. Future samples will have no memory of our sample.

16.24 The mistake is in saying that 95% of other polls would have results close to the results of this poll. Other surveys should be close to the truth—not necessarily close to the results of this survey. (Additionally, there is the suggestion that 95% means "exactly 19 out of 20" when, really, 95% refers to repeating the survey infinitely often.)

16.25 (a) A stemplot of the data is provided. The distribution has a noticeable left-skew. The data do not appear to follow a Normal distribution.

Stem and Leaf

Stem	Leaf	Count
33	0237	4
32	033677	6
31	399	3
30	259	3
29		
28	7	1
27		
26	5	1
25		
24	1	1
23	0	1

23|0 represents 23000

(b) STATE: What is the mean load μ required to pull apart pieces of Douglas fir? PLAN: We will estimate μ by giving a 95% confidence interval. SOLVE: The problem states that we are willing to take this sample to be an SRS of the population. In spite of the shape of the stemplot, we are told to assume that this distribution is Normal, with standard deviation σ = 3000 pounds. We find $\bar{x}$ = 30,841 pounds, so the 95% confidence interval for μ is given by $30{,}841 \pm 1.96\frac{3000}{\sqrt{20}} = 30{,}841 \pm 1314.81 = 29{,}526.19$ to $32{,}155.81$ pounds. CONCLUDE: With 95% confidence, the mean load μ required to break apart pieces of Douglas fir is between 29,526.2 and 32,155.8 pounds. However, given the shape of the distribution of the data, we cannot rely much on this interval.

16.26 (a) The provided stemplot shows that the data look reasonably Normal.

−8	3
−7	80
−6	88552
−5	97633221
−4	9977430
−3	86310
−2	755322110
−1	800
−0	83
0	234
1	7
2	2

(b) STATE: What is the mean percent change μ in spinal mineral content of nursing mothers? PLAN: We will estimate μ by giving a 99% confidence interval. SOLVE: The problem states that we may consider these women to be an SRS of the population. In part (a), we concluded that the data appear as though they may have come from a Normal distribution. We find $\bar{x}$ = −3.587%, so the 99% confidence interval for μ is given by $-3.587\% \pm 2.576\frac{2.5\%}{\sqrt{47}} = -3.587\% \pm 0.939\% = -4.526\%$ to -2.648%. CONCLUDE: We are 99% confident that the mean percent change in spinal mineral content of nursing mothers is between −4.526% and −2.648%. That is, the mean spinal mineral content in nursing mothers decreases by between 2.65% and 4.53% with 99% confidence.

16.27 (a) A stemplot is provided. There is little evidence that the sample does not come from a Normal distribution. For inference, we must assume that the 10 untrained students were selected randomly from the population of all untrained people.

Stem and Leaf

Stem	Leaf	Count
4	2	1
3	5	1
3	0013	4
2	9	1
2	23	2
1	9	1

1|9 represents 19

(b) STATE: What is the average (mean) DMS odor threshold, μ, for all untrained people? PLAN: We will estimate μ with a 95% confidence interval. SOLVE: We have assumed that we have a random sample and that the population from which we are sampling is Normal. We obtain $\bar{x}$ = 29.4 μg/L. Our 95% confidence interval for μ is given by $29.4 \pm 1.96\frac{7}{\sqrt{10}} =$

$29.4 \pm 4.34 = 25.06$ to 33.74 μg/L. CONCLUDE: With 95% confidence, the mean sensitivity for all untrained people is between 25.06 and 33.74 μg/L.

16.28 Regardless of the level of confidence (the 95% confidence level has nothing to do with it), larger samples reduce margins of error, which provides greater precision in estimating μ.

16.29 and **16.30** are Web-based exercises.