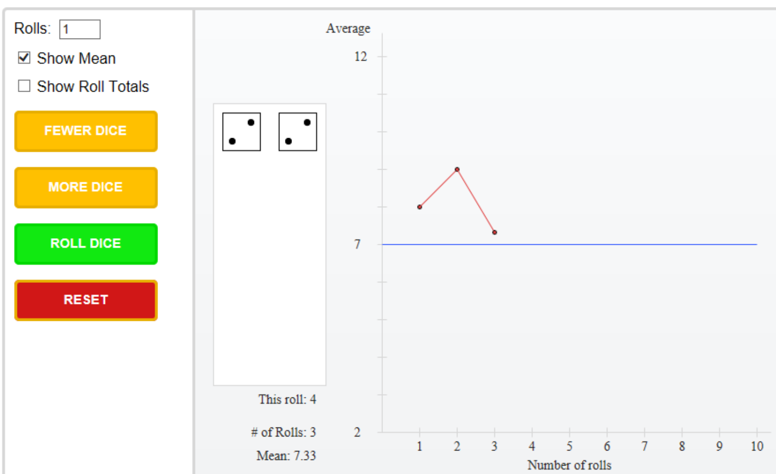
Chapter 15 Solutions

15.1 Both are statistics (they came from the 11 subjects in the experiment).

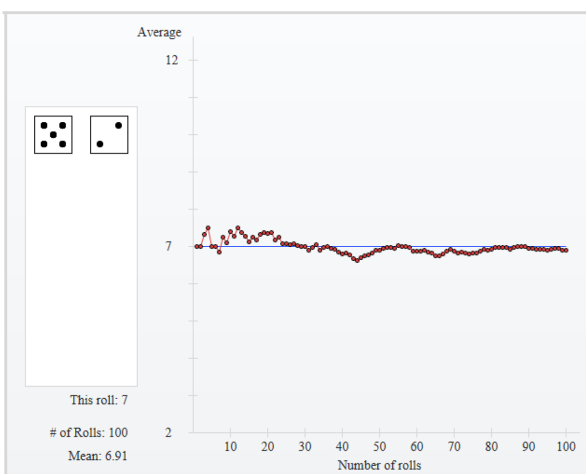
15.2 38% and 36% are parameters (they are based on all registered voters in the state); 35% came from the randomly chosen 250 telephone calls, so it is a statistic.

15.3 27.2% and 4.5% are statistics (they are based on the survey of 15,624 American high school students); 27.5% is a parameter (it is based on all American high school students).

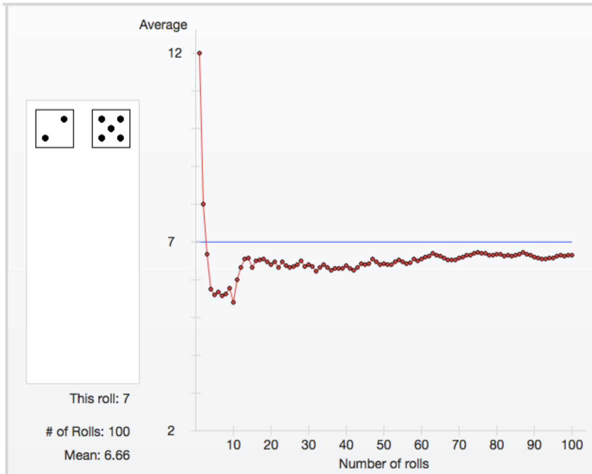
15.4 (Results will vary.) **(a)** A sample output screen is provided. One realization had rolls of 8, 10, and 4. After three rolls, the mean was 7.33.



(b) One realization is given, with a total of 100 rolls. This simulation ended with a mean of 6.91. After 100 sets of rolls, approximately 95% of students will have a mean between 5.5 and 8.5.



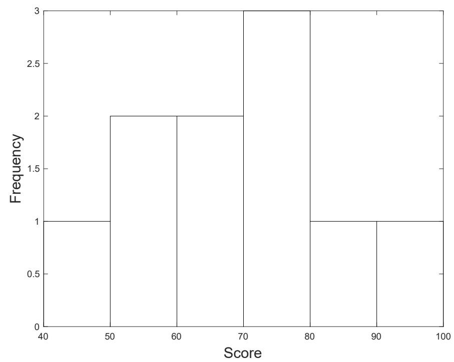
(c) A second realization is given, with a final mean of 6.66. This simulation graph is very different from the graph given for part (b), but in both realizations the average eventually gets close to the population mean $\mu = 7$.



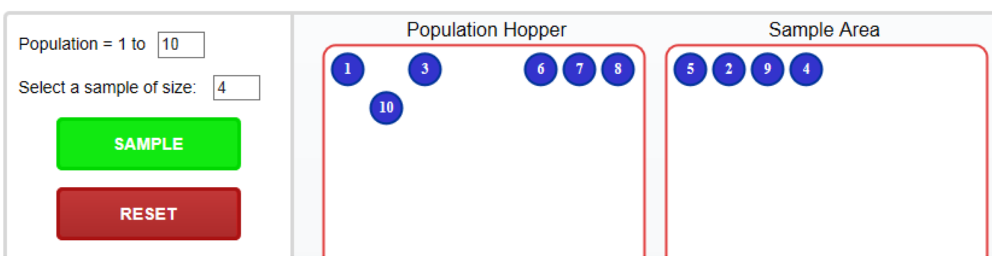
15.5 Although the probability of having to pay for a total loss for one or more of the 10 policies is very small, if this were to happen, it would be financially disastrous. On the other hand, for thousands of policies, the law of large numbers says that the average claim on many policies will be close to the mean, so the insurance company can be assured that the premiums it collects will (almost certainly) cover claims.

15.6 (a) The population is the 10,900 respondents to the American Time Use Survey; the population distribution (Normal with mean 529.9 minutes and standard deviation 135.6 minutes) describes the minutes of sleep per night for the individuals in this population. **(b)** The sampling distribution (Normal with mean 529.9 minutes and standard deviation 13.56 minutes) describes the distribution of the average sleep time for 100 randomly selected individuals from this population.
Note: *Students may say that the population is all Americans. That is the population from which the individuals who responded to the survey were drawn, but because we are sampling again from those respondents, the respondents become the population of interest.*

15.7 (a) The histogram is provided.



(b) The mean is μ = 67.7. **(c)** and **(d)** Results will vary. The result of one sample is shown. This sample selects students 5, 2, 9, and 4 with scores of 72, 63, 75, and 45. Their mean is $\bar{x} = \frac{72 + 63 + 75 + 45}{4} = 63.75$. Students should repeat this process until they have 10 sample means.



15.8 (a) "Unbiased" means that, on average (in the long run), the value of $\bar{x}$ is the same as the population mean, μ. It does not address individual sample results. **(b)** Large samples are more trustworthy because more of the population is represented in the samples. The sample mean of a large sample will be closer to μ by the law of large numbers.

15.9 (a) The sampling distribution of $\bar{x}$ is $N\left(182, \frac{37}{\sqrt{100}}\right) = N(182 \text{ mg/dL}, 3.7$ mg/dL). Therefore, $P(180 < \bar{x} < 184) = P\left(\frac{180-182}{3.7} < Z < \frac{184-182}{3.7}\right) =$ $P(-0.54 < Z < 0.54) = 0.7054 - 0.2946 = 0.4108$, using Table A. **(b)** With n = 1000, the sampling distribution of $\bar{x}$ is $N(182 \text{ mg/dL}, 1.17 \text{ mg/dL})$, so $P(180 < \bar{x} < 184) = P\left(\frac{180-182}{1.17} < Z < \frac{184-182}{1.17}\right) = P(-1.71 < Z < 1.71) =$ $0.9564 - 0.0436 = 0.9128$, using Table A.

15.10 (a) $\frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{4}} = 5$ mg. **(b)** Solve $\frac{\sigma}{\sqrt{n}} = 2$, or $\frac{10}{\sqrt{n}} = 2$, so $\sqrt{n} = 5$, or $n = 25$. The average of several measurements is more likely than a single measurement to be close to the mean.

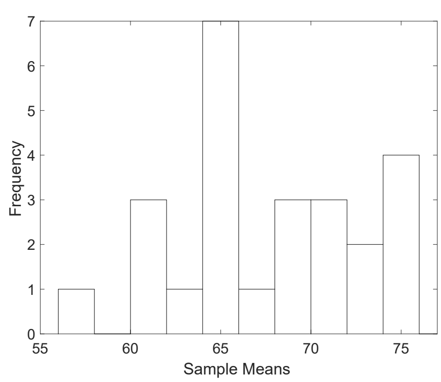
15.11 No. The histogram of the sample values will look like the population distribution, whatever it might happen to be. (For example, if we roll a fair die many times, the histogram of sample values should look relatively flat—probability close to 1/6 for each value 1, 2, 3, 4, 5, and 6.) The central limit theorem says that the histogram of sample means (from many large samples) will look more and more Normal.

15.12 (a) The mean and standard deviation of $\bar{x}$ are 2.4 and $\frac{3.7}{\sqrt{50}} = 0.52$ ash borers, respectively. **(b)** Because this distribution is only approximately Normal, it would be quite reasonable to use the 68–95–99.7 rule to give a rough estimate: 3.0 is about 1 standard deviation above the mean, so the probability should be about 0.16 (half of the 32% that falls outside ±1 standard deviation). Alternatively, $P(\bar{x} > 3.0) = P\left(Z > \frac{3 - 2.4}{0.52}\right) = P(Z > 1.15) = 1 - 0.8749 = 0.1251$.

15.13 STATE: We ask, what is the probability that the average loss for 10,000 such policies will be no greater than $135, when the long-run average loss is $130? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that, in spite of the skewness of the population distribution, the average loss among 10,000 policies will be approximately $N\left(\$130, \frac{\$300}{\sqrt{10{,}000}}\right) = N(\$130, \$3)$. Now, $P(\bar{x} \le \$135) = P\left(Z \le \frac{135 - 130}{3}\right) = P(Z \le 1.67) = 0.9525$. CONCLUDE: We can be about 95.25% certain that average losses will not exceed $135 per policy.

15.14 A sample variance of 9.2 (s = 3.033) is on the low end of the histogram shown in Figure 15.7 (b) of the text. However, because it is not unusual, we cannot say that the variability in the palates of these senior citizens is different from the rest of the population.

15.15 (Answers will vary due to randomness.) **(a)** One repetition of 25 sample means resulted in the histogram provided.



(b) According to the histogram in part (a), each sample mean is less than 76 and the chance of obtaining $\bar{x} \ge 78$ is very unlikely. **(c)** The mean for these four students is

78. Because a sample mean of 78 (or higher) never occurred, we might conclude that the mean score for the four honors students was unusually high (that is, this mean is statistically significant).
Note: *Taking 1000 samples of size 4 resulted in* $\bar{x} \geq 78$ *only 36 times. Thus, we can conclude that the estimated probability of obtaining a sample mean at least as big as 78 is 0.036.*

15.16 (b) statistic. This is a proportion of the people interviewed in the sample of 60,000 households.

15.17 (c) parameter. 68.3% is a proportion of all registered voters (the population).

15.18 (b) as you invest in more and more stocks chosen at random, your average return on these stocks gets ever closer to 11.2%. The law of large numbers says that the mean from a large sample is close to the population mean. Note that option (c) is also true, but it is based on the central limit theorem, not on the law of large numbers.

15.19 (a) 495. The mean of the sample means ($\bar{x}$ values) is the same as the population mean (μ).

15.20 (c) $118/\sqrt{100}$ = 11.8. The standard deviation of the distribution of $\bar{x}$ is $\sigma/\sqrt{n}$.

15.21 (a) in many samples from this population, the mean of the many values of $\bar{x}$ will be equal to μ. “Unbiased” means that the estimator is right on the average.

15.22 (c) the average failure time of a large number of batteries has a distribution that is close to Normal. The central limit theorem says that the mean from a large sample has (approximately) a Normal distribution. Note that option (a) is also true, but it is based on the law of large numbers, not on the central limit theorem.

15.23 (b) 0.27. For n = 6 women, $\bar{x}$ has an $N\left(266, \frac{16}{\sqrt{6}}\right) = N(266, 6.5320)$ distribution, so $P(\bar{x} > 270) = P(Z > 0.61) = 0.2709$.

15.24 1 is a parameter (the mean of the population of all conductivity measurements); 1.07 is a statistic (the mean of the 10 measurements in the sample).

15.25 Both 25.40 and 20.41 are statistics (related, respectively, to the two samples).

15.26 In the long run, the gambler earns an average of 94.7 cents per bet. In other words, the gambler loses (and the house gains) an average of 5.3 cents for each $1 bet.

15.27 (a) $P(495 < X < 505) = P\left(\frac{495 - 500}{10.4} < Z < \frac{505 - 500}{10.4}\right) = P(-0.48 < Z <$

$0.48) = 0.6844 - 0.3156 = 0.3688$. **(b)** If n = 25 students, the sampling distribution of $\bar{x}$ is $N(500, 10.4/\sqrt{25}) = N(500, 2.08)$. **(c)** $P(495 < X < 505) = P\left(\frac{495-500}{2.08} < Z < \frac{505-500}{2.08}\right) = P(-2.40 < Z < 2.40) = 0.9918 - 0.0082 = 0.9836$.

15.28 (Let X be Shelia's measured glucose level.) **(a)** $P(X > 130) = P(Z > 0.67)$ = 0.2514. **(b)** If $\bar{x}$ is the mean of four measurements (assumed to be independent), then $\bar{x}$ has an $N(122, 12/\sqrt{4})$ = N(122 mg/dl, 6 mg/dl) distribution, and $P(\bar{x} > 130)$ = $P(Z > 1.33)$ = 0.0918.

15.29 **(a)** Let $\bar{x}$ be the mean number of minutes per day that the five randomly selected, mildly obese people spend walking. Then $\bar{x}$ has the $N(373, 67/\sqrt{5})$ = N(373 minutes, 29.96 minutes) distribution. Now $P(\bar{x} > 420) = P\left(Z > \frac{420-373}{29.96}\right) = P(Z > 1.57) = 0.0582$. **(b)** Let $\bar{x}$ be the sample mean number of minutes per day for the five randomly selected lean people. $\bar{x}$ has the $N(526, 107/\sqrt{5})$ = N(526 minutes, 47.85 minutes) distribution. $P(\bar{x} > 420) = P(Z > -2.22) = 0.9868$.

15.30 As shown in Exercise 15.28(b), the mean of four measurements has an N(122 mg/dl, 6 mg/dl) distribution, and $P(Z > 1.645) = 0.05$ if Z is $N(0, 1)$, so $L = 122 + 1.645 \times 6 = 131.87$ mg/dl.

15.31 **(a)** For the emissions X of a single car, $P(X > 86) = P\left(Z > \frac{86-80}{4}\right) = P(Z > 1.5)$ = 1 – 0.9332 = 0.0668. **(b)** The average $\bar{x}$ has the $N\left(80 \text{ mg/mi}, \frac{4}{\sqrt{25}} \text{ mg/mi}\right)$ = $N(80 \text{ mg/mi}, 0.8 \text{ mg/mi})$ distribution. Therefore, $P(\bar{x} > 86) = P\left(Z > \frac{86-80}{0.8}\right) = P(Z > 7.5)$, which is essentially 0.

15.32 **(a)** $\bar{x}$ will have an approximately Normal distribution, with mean 8.8 beats per five seconds and standard deviation $1/\sqrt{24}$ = 0.204124 beats per five seconds. **(b)** $P(\bar{x} < 8) = P(Z < -3.92)$, which is essentially 0. **(c)** If the total number of beats in one minute is less than 100, then the average over 12 five-second intervals needs to be less than 100/12 = 8.333 beats per five seconds. $\bar{x}$ will have an approximately Normal distribution, with mean 8.8 beats per five seconds and standard deviation $1/\sqrt{12}$ = 0.288675 beats per five seconds. $P(\bar{x} < 8.333) = P(Z < -1.62) = 0.0526$.

15.33 The mean NOX level for 25 cars has an N(80 mg/mi, 0.8 mg/mi) distribution, and $P(Z > 2.33) = 0.01$ if Z is $N(0, 1)$, so $L = 80 + (2.33)(0.8) = 81.864$ mg/mi.

15.34 STATE: What is the probability of an average return over 10%? What is the probability of an average return less than 5%? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that over 40 years, $\bar{x}$ (the mean return) is approximately Normal, with mean μ = 11.0% and standard deviation $17.0\%/\sqrt{40}$ = 2.688%. Thus, $P(\bar{x} > 10\%) = P(Z > -0.37) = 1 -$

0.3557 = 0.6443, and $P(\bar{x} < 5\%) = P(Z < -2.23) = 0.0129$. CONCLUDE: There is about a 64% chance of getting average returns over 10% and a 1% chance of getting average returns less than 5%.
Note: *We have to assume that returns in separate years are independent.*

15.35 STATE: What is the probability that the total weight of the 22 passengers exceeds 4500 pounds? PLAN: Use the central limit theorem to approximate this probability. SOLVE: If W is total weight, then the sample mean weight is $\bar{x} = W/22$. The event that the total weight exceeds 4500 pounds is equivalent to the event that $\bar{x}$ exceeds 4500/22 = 204.55 pounds. The central limit theorem says that $\bar{x}$ is approximately Normal, with mean 195 pounds and standard deviation $35/\sqrt{22}$= 7.462 pounds. Therefore, $P(W > 4500) = P(\bar{x} > 204.55) = P\left(Z > \frac{204.55 - 195}{7.462}\right) = P(Z > 1.28) = 0.1003$. CONCLUDE: There is about a 10% chance that the total weight exceeds 4500 pounds.

15.36 We need to choose n so that $10.4/\sqrt{n} = 1$. That means $\sqrt{n} = 10.4$, so n = 108.16. Because n must be a whole number, take n = 109.

15.37 (a) 99.7% of all observations fall within 3 standard deviations, so we want $3\sigma/\sqrt{n} = 1$. The standard deviation of $\bar{x}$ $(\sigma/\sqrt{n})$ must therefore be 1/3 point. **(b)** We need to choose n so that $10.4/\sqrt{n}= 1/3$. This means $\sqrt{n} = 31.2$, so n = 973.44. Because n must be a whole number, take n = 974.

15.38 On average, Joe loses 40 cents each time he plays (that is, he spends $1 and gets back 60 cents).

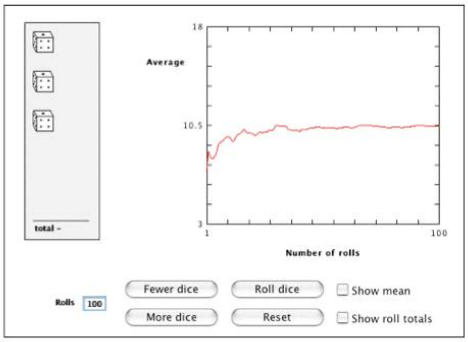
15.39 (a) With n = 14,000, the mean and standard deviation of $\bar{x}$ are $0.60 and $\$18.96/\sqrt{14{,}000} = \$0.1602$, respectively. **(b)** $P(\$0.50 < \bar{x} < \$0.70) = P(-0.62 < Z < 0.62) = 0.4648$.

15.40 (a) With n = 150,000, the mean and standard deviation of $\bar{x}$ are $0.40 and $\$18.96/\sqrt{150{,}000} = \$0.0490$, respectively. **(b)** $P(\$0.30 < \bar{x} < \$0.50) = P(-2.04 < Z < 2.04) = 0.9586$.

15.41 (a) The estimate in Exercise 15.39(b) was 0.4648, so the Normal approximation underestimates the exact answer by about 0.03. **(b)** With n = 3500, the Normal approximation gives $P(\$0.50 < \bar{x} < \$0.70) = P(-0.31 < Z < 0.31)$, which is 0.2434 (Table A). This is quite a bit smaller than the exact answer. **(c)** The probability that their average winnings fall between $0.50 and $0.70 is the same as the probability found in Exercise 15.40(b), for which the Normal approximation gives 0.9586, so the approximation differs from the exact value by only about 0.004.
Note: *Averaging between $0.50 and $0.70 per bet means winning a total of between $0.5*n *and $0.7*n*. With 14,000 bets, this means winning between $7000 (11.667 actual wins) and $9800 (16.333 wins). In the 3500 bets case, the result is winning between*

$1750 and $2450, which represents 2.91 and 4.08 (essentially 3 or 4) wins. This is not a large number, as we saw in Chapter 14 (if it was covered) and will see in Chapter 20.

15.42 The mean is 10.5 = (3)(3.5) because a single die has a mean of 3.5. Sketches will vary, as will the number of rolls; one result is shown.



15.43 The probability found in Exercise 12.50 was 1/20 = 0.05. Because a 5% probability is fairly small, we could consider this result statistically significant.

15.44 and **15.45** are Web-based exercises.