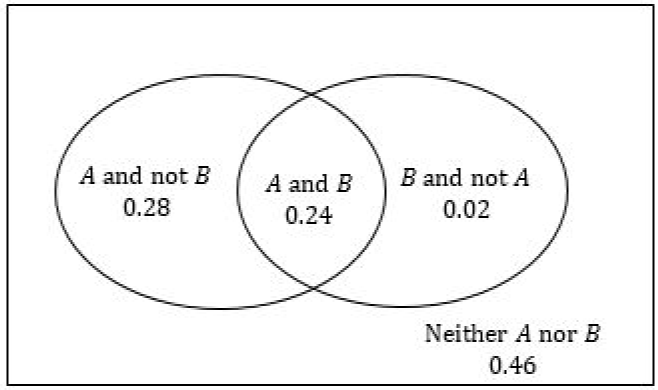
Chapter 13 – General Rules of Probability

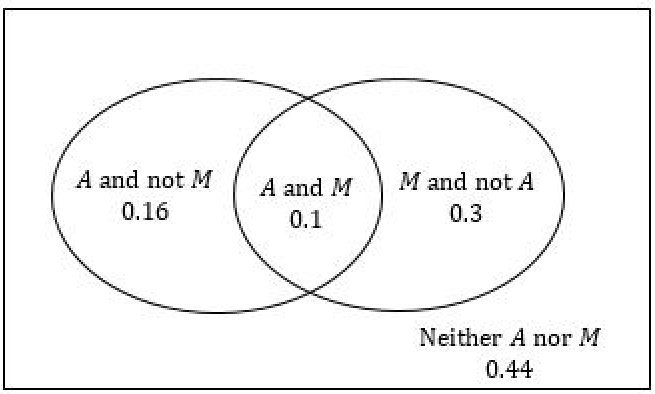
13.1 (a) A Venn diagram is provided. Because $P(A \text{ or } B) = 0.54 = P(A) + P(B) - P(A \text{ and } B) = 0.52 + 0.26 - P(A \text{ and } B)$, we must have $P(A \text{ and } B) = 0.52 + 0.26 - 0.54 = 0.24$.

(b) Event (A and B) is a user who posts both photos and videos that he or she has taken. Event (A and not B) is a user who posts photos that he or she has taken, but not videos. Event (B and not A) is a user who posts videos that he or she has taken, but not photos.

Event (neither A nor B) is a user who posts neither photos nor videos that he or she has taken. **(c)** $P(A \text{ and } B) = 0.24$; $P(A \text{ and not } B) = 0.52 - 0.24 = 0.28$; $P(\text{not } A \text{ and } B) = 0.26 - 0.24 = 0.02$; and $P(\text{neither } A \text{ nor } B) = 1 - P(A \text{ or } B) = 1 - 0.54 = 0.46$. See the diagram in part (a).



13.2 [A Venn diagram is provided. A is the event the degree is an associate degree, and M is the event the degree was earned by a man. The probability of both A and M occurring is given. Subtracting this from the given probabilities for A and M gives the probabilities of the rest of those events. Those probabilities add to 0.56, so $P(\text{neither } A \text{ nor } W) = 0.44$.]



(a) $P(A \text{ and not } M) = 0.16$, or 16%. **(b)** P(woman earns degree other than associate degree) = 0.44/0.6 = 0.73, or 73%. **(c)** $P(\text{not } M) = P(A \text{ and not } M) + P(\text{not } A \text{ and not } M) = 0.16 + 0.44 = 0.6$, or 60%.

13.3 It is unlikely that these events are independent. In particular, it is reasonable to expect that younger adults are more likely than older adults to be college students. *[**Note:** Using the notation of conditional probability introduced later in this chapter, we believe* P(*college student* | *over 55*) < *0.08.]*

13.4 This would not be very surprising, assuming that all the authors are independent (for example, none were written by siblings or married couples), we can view the nine names as being a random sample, and the probability that none of these are among the ten most common names is $P(N = 0) = (1 - 0.096)^9 = 0.4032$.

13.5 If we assume that each site is independent of the others (and that they can be considered as a random sample from the collection of sites referenced in scientific journals), then P(all seven are still good) = $(1-0.13)^7 = (0.87)^7 = 0.3773$.

13.6 $P(M|A) = \frac{P(M \text{ and } A)}{P(A)} = \frac{0.1}{0.26} = 0.385$.

13.7 Using the Venn diagram from Exercise 13.1: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.24}{0.26} = 0.923$.

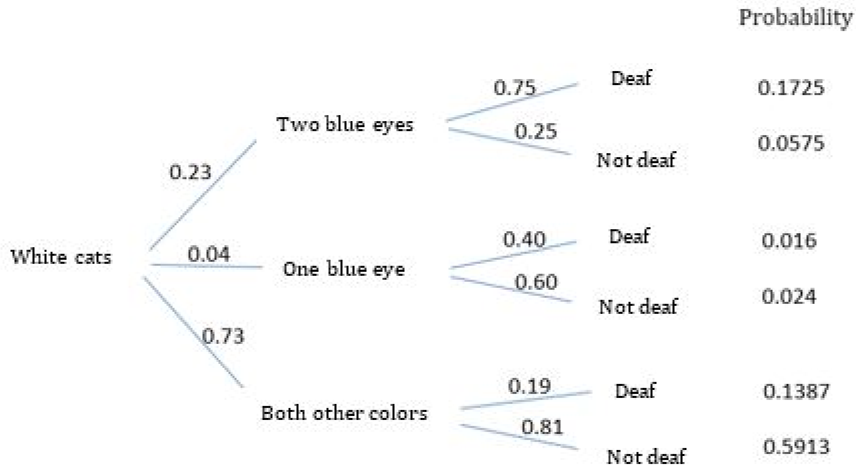
13.8 Let R be the event game is a role-playing game, while S is the event game is a strategy game. Then $P(\text{not } S) = 1 - 0.354 = 0.646$, and $P(R \mid \text{not } S) = \frac{P(R \text{ and not } S)}{P(\text{not } S)} = \frac{0.139}{0.646} = 0.2152$.

13.9 Let H be the event that an adult belongs to a club, and T be the event that he or she goes at least twice a week. We have been given $P(H) = 0.08$ and $P(T \mid H) = 0.45$. Note also that $P(T \text{ and } H) = P(T)$, because one has to be a member of the club in order to attend. So $P(T) = P(H)P(T \mid H) = (0.08)(0.45) = 0.036$.

13.10 STATE: We want to calculate the percent of all teens who are online, use social media, and have posted a picture of themselves. PLAN: Express the information we are given in terms of events and their probabilities. Let A = {the teen is online}, B = {the teen uses social media}, and C = {the teen has posted a picture of himself or herself}. Then $P(A) = 0.95$, $P(B \mid A) = 0.81$, and $P(C \mid A \text{ and } B) = 0.91$. We want to find $P(A \text{ and } B \text{ and } C)$. SOLVE: Use the multiplication rule $P(A \text{ and } B \text{ and } C) = P(A)P(B \mid A)P(C \mid A \text{ and } B) = (0.95)(0.81)(0.91) = 0.700$. CONCLUDE: 70% of all teens are online, use social media, and have posted a picture of themselves.

13.11 **(a)** Let W be the event that a professor is a woman. $P(W) = \frac{87 + 137 + 243}{222 + 324 + 972} = \frac{467}{1518} = 0.308$. **(b)** Let F be the event that the professor is a full professor. Then $P(W|F) = \frac{P(W \text{ and } F)}{P(F)} = \frac{243}{972} = 0.25$. **(c)** No, rank and sex are not independent because $P(W) \neq P(W \mid F)$. The probability of being a woman is different for each rank.

13.12 (a) A tree diagram is below.



(b) $P(\text{deaf}) = P(\text{deaf} \mid \text{two blue})P(\text{two blue}) + P(\text{deaf} \mid \text{one blue})P(\text{one blue}) +$ $P(\text{deaf} \mid \text{no blue})\ P(\text{no blue}) = 0.1725 + 0.016 + 0.1387 = 0.3272.$

13.13 (a) $P(\text{Age 18–24} \mid \text{online date no}) = \frac{P(\text{Age 18–24 and online date no})}{P(\text{online date no})} = \frac{0.1241}{0.1241 + 0.3198 + 0.3654} = \frac{0.1241}{0.8093} = 0.1533$; $P(\text{Age 25–44} \mid \text{online date no}) = \frac{0.3198}{0.8093} = 0.3952$; and $P(\text{Age 45–64} \mid \text{online date no}) = \frac{0.3654}{0.8093} = 0.4515$. **(b)** We see the conditional probability for both the 18–24 age group and the 25–44 age group is lower than the unconditional probability, and it is higher for the 45–64 age group. This is what one would expect, since younger adults are more likely to use online dating than older adults.

13.14 (a) $P(\text{two blue} \mid \text{deaf}) = \frac{P(\text{two blue and deaf})}{P(\text{deaf})} = \frac{0.1725}{0.3272} = 0.5272$; $P(\text{one blue} \mid \text{deaf}) = \frac{0.016}{0.3272} = 0.0489$; and $P(\text{other colors} \mid \text{deaf}) = \frac{0.1387}{0.3272} = 0.4239$. Observe that 0.5272 + 0.0489 + 0.4239 = 1. **(b)** $P(\text{two blue and deaf}) = 0.1725$. Observe that $P(\text{two blue})P(\text{deaf}) = (0.23)(0.3272) = 0.0753 \neq 0.1725$. Thus, having two blue eyes and being deaf are not independent events.

13.15 Using the notation defined in Example 13.14, $P(B_1 | \text{not } A) = \frac{P(\text{not } A \mid B_1)P(B_1)}{P(\text{not } A \mid B_1)\ P(B_1) + P(\text{not } A \mid B_2)P(B_2)} = \frac{(1-0.21)(0.063)}{(1-0.21)(0.063) + (1-0.06)(1-0.063)} = \frac{0.0498}{0.0498 + 0.8808} = 0.0535.$

13.16 (a) Using the notation from Example 13.14, $P(B_2 \mid A) = \frac{P(A \mid B_2)P(B_2)}{P(A \mid B_2)\ P(B_2) + P(A \mid B_1)P(B_1)} = \frac{(0.06)(1-0.03)}{(0.06)(1-0.03) + (0.21)(0.03)} = 0.9023$. This probability is larger than it was when 6.3% of the population had the disease. This has increased because if a person is less likely to actually have the disease, then the probability (conditional or unconditional) of not having the disease will increase, provided everything else stays the same. **(b)** As the rate of disease decreases, the false-positive rate increases. This implies that screening for a very rare disease will have a high false-positive rate.

13.17 (a) The prior probabilities of black, brown, blonde, and red hair are 0.061, 0.461, 0.453, and 0.025, respectively. **(b)** First, observe that $P(\text{blue}) = (0.061)(0.03) + (0.461)(0.259) + (0.453)(0.562) + (0.025)(0.473) = 0.3876$. So, $P(\text{black} \mid \text{blue}) = \frac{P(\text{black and blue})}{P(\text{blue})} = \frac{(0.061)(0.03)}{0.3876} = 0.0047$; $P(\text{brown} \mid \text{blue}) = \frac{(0.461)(0.259)}{0.3876} = 0.3080$; $P(\text{blonde} \mid \text{blue}) = \frac{(0.453)(0.562)}{0.3876} = 0.6568$; and $P(\text{red} \mid \text{blue}) = \frac{(0.025)(0.473)}{0.3876} = 0.0305$. The probabilities are what one would expect. People with blonde and red hair had the highest probability of having blue eyes, so we would expect the conditional probabilities for those two groups to be larger than the prior probabilities.

13.18 (b) 0.94. This probability is $(1 - 0.02)^3 = (0.98)^3 = 0.9412$.

13.19 (b) 0.06. We use $P(\text{at least 1}) = 1 - P(\text{none})$, so the probability is $1 - (0.98)^3 = 0.0588$.

13.20 (a) 0.98. $P(\text{at least one positive}) = 1 - P(\text{both negative}) = 1 - P(\text{first negative})P(\text{second negative}) = 1 - (0.1)(0.2) = 0.98$. Because the tests are independent, we can multiply the probabilities of a negative test on each.

13.21 (b) 0.34. This probability is $P(\text{other}) = \frac{1017}{2977} = 0.3416$.

13.22 (b) 0.25. This probability is $P(2\text{ year} \mid \text{male}) = \frac{374}{1524} = 0.2454$.

13.23 (c) 0.47. This probability is $P(\text{female} \mid 4\text{ year}) = \frac{658}{1252} = 0.5256$.

13.24 (b) $P(A \mid B)$.

13.25 (b) 0.66. $P(W \text{ or } NM) = P(W) + P(NM) - P(W \text{ and } NM) = 0.52 + 0.25 - 0.11 = 0.66$.

13.26 (c) 0.024. $P(W \text{ and } D) = P(W)P(D \mid W) = (0.86)(0.028) = 0.024$.

13.27 (b) 0.030. $P(D) = P(W \text{ and } D) + P(B \text{ and } D) + P(A \text{ and } D) = (0.86)(0.028) + (0.12)(0.044) + (0.02)(0.035) = 0.030$.

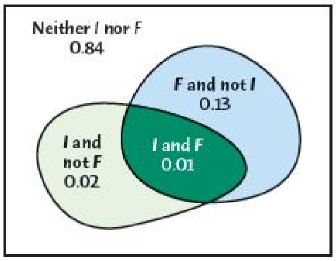
13.28 $P(8\text{ losses}) = (1 - 0.25)^8 = (0.75)^8 = 0.1001$.

13.29 $P(\text{none are O-negative}) = (1 - 0.072)^{10} = 0.4737$, so $P(\text{at least one is O-negative}) = 1 - 0.4737 = 0.5263$.

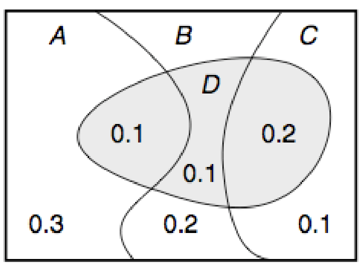
13.30 (a) $P(\text{win the jackpot}) = (1/20)(9/20)(1/20) = 0.001125$. **(b)** The other (non-cherry) symbol can show up on the middle wheel, with probability $(1/20)(11/20)(1/20) = 0.001375$, or on either of the outside wheels, with probability $= (19/20)(9/20)(1/20)$ each. **(c)** Combining all three cases from part (b), we have $P(\text{exactly two cherries}) =$

0.001375 + 2(0.021375) = 0.044125.

13.31 PLAN: Let I be the event infection occurs and let F be the event repair fails. We have been given $P(I)$ = 0.03, $P(F)$ = 0.14, and $P(I$ and $F)$ = 0.01. We want to find $P($not I and not $F)$. SOLVE: First use the general addition rule $P(I$ or $F) = P(I) + P(F) - P(I$ and $F)$ = 0.03 + 0.14 − 0.01 = 0.16. This is the shaded region in the Venn diagram provided. Now observe that the desired probability is the complement of I or F (the unshaded region); $P($not I and not $F)$ = 1 − $P(I$ or $F)$ = 0.84. CONCLUDE: 84% of operations succeed and are free from infection.



13.32 Note that in this diagram, events A, B, and C should not overlap and should account for all possibilities (that is, those three events fill the entire diagram). Meanwhile, D intersects all three of the others. The probabilities $P(A$ and $D)$, $P(B$ and $D)$, and $P(C$ and $D)$ give the probability of each overlapping region, and the portion of each event A, B, and C outside of D must account for the rest of that event's probability. As can be seen from the diagram, $P(D)$ = 0.4 = 0.1 + 0.1 + 0.2.



13.33 PLAN: Let I be the event infection occurs and let F be the event repair fails. Refer to the Venn diagram in Exercise 13.31 (ignoring the shading). We want to find $P(I \mid \text{not } F)$. SOLVE: We have $P(I \mid \text{not } F) = \frac{P(I \text{ and not } F)}{P(\text{not } F)} = \frac{0.02}{0.86}$ = 0.0233. CONCLUDE: The probability of infection given that the repair is successful is 0.0233. That is, in 2.33% of all successful operation cases, the patient develops an infection.

13.34 (a) P(positive | disease) = 564/574 = 0.9826. **(b)** P(no disease | negative) = 708/718 = 0.9861.

13.35 (a) $P(\$25{,}000 \text{ or more}) = 0.184 + 0.101 + 0.024 + 0.018 = 0.327$. **(b)** $P(\text{at least } \$100{,}000 \mid \text{at least } \$25{,}000) = \frac{P(\text{at least } \$100{,}000)}{P(\text{at least } \$25{,}000)} = \frac{0.024 + 0.018}{0.327} = 0.1284.$

13.36 (a) P(two boys | at least one boy) = $\frac{P(\text{two boys})}{P(\text{at least one boy})} = \frac{0.25}{0.75} = \frac{1}{3}$. **(b)** P(two boys | older child is a boy) = $\frac{P(\text{two boys})}{P(\text{older is a boy})} = \frac{0.25}{0.50} = \frac{1}{2}$. Note that we can also find this by reasoning that *P*(two boys | older child is a boy) = *P* (younger child is a boy | older child is a boy). Because the two children's sexes are independent, this probability is the same as the unconditional probability *P*(younger child is a boy) = 0.5.

13.37 (Let *W* be the event the person is a woman and *M* be the person earned a Master's degree.) **(a)** *P*(not *W*) = 1758/4345 = 0.4046. **(b)** *P*(not *W* | *M*) = 349/938 = 0.3721. **(c)** The events choose a man and choose a Master's degree recipient are not independent. If they were, the two probabilities in part (a) and part (b) would be equal.

13.38 (a) The probability the child has blue eyes if he or she has red hair is *P*(blue eyes | red hair) = 0.473; *P*(blue eyes and red hair) = (0.025)(0.473) = 0.0118. **(b)** The probability of having freckles if the child has both red hair and blue eyes is *P*(freckles | red hair and blue eyes) = 0.857; *P*(red hair and blue eyes and freckles) = (0.025)(0.473)(0.857) = 0.0101.

13.39 (Let *W* be the event the person is a woman and *M* be the person earned a Master's degree.) **(a)** *P*(*W*) = 2587/4345 = 0.5954. **(b)** *P*(associate's degree | *W*) = 756/2587 = 0.2922. **(c)** Using the multiplication rule: *P*(female and associate's degree) = *P*(associate's degree | *W*)*P*(*W*) = (0.2922)(0.5954) = 0.174. Using the table, we see this probability is 756/4345 = 0.174.

13.40 (a) *P*(red hair) = 0.025 is the probability a child has red hair, regardless of eye color or freckle status. *P*(blue eyes) = (0.061)(0.030) + (0.461)(0.259) + (0.453)(0.562) + (0.025)(0.473) = 0.3876 is the probability a child has blue eyes, regardless of hair color or freckle status. *P*(freckles) = 0.061((0.030)(0.143) + (0.047)(0.182) + (0.923)(0.018)) + 0.461((0.259)(0.278) + (0.327)(0.232) + (0.414)(0.153)) + 0.453((0.562)(0.319) + (0.315)(0.302) + (0.123)(0.164)) + 0.025((0.473)(0.857) + (0.405)(0.767) + (0.122)(0.778)) = 0.2529 is the probability a child has freckles, no matter what the hair or eye color. **(b)** *P*(freckles and red hair) = 0.0203 is the probability the child has red hair and freckles regardless of the eye color; it was the last row of the probability found in part (a). *P*(freckles | red hair) = 0.0203/0.025 = 0.812 is the probability of freckles if the child has red hair and eye color is ignored. **(c)** Because *P*(freckles | red hair) = 0.812 is not equal to *P*(freckles) = 0.2529, freckles and hair color are not independent in the population of Germans of Caucasian descent.

13.41 (a) and **(b)** These probabilities are provided in the table.

P(1st card is ♠)	13/52	= 0.25
P(2nd card is ♠ | first card is ♠)	12/51	= 0.2353
P(3rd card is ♠ | first two cards are ♠)	11/50	= 0.22
P(4th card is ♠ | first three cards are ♠)	10/49	= 0.2041
P(5th card is ♠ | first four cards are ♠)	9/48	= 0.1875

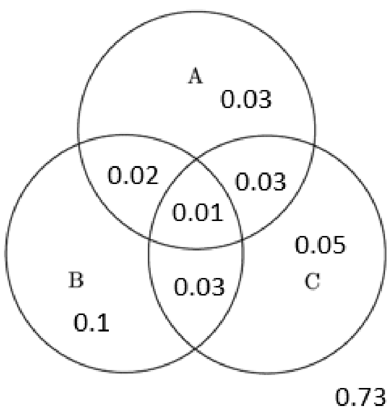
(c) The product of these conditional probabilities gives the probability of a flush in spades by the general multiplication rule; we must draw a spade, and then another, and then a third, a fourth, and a fifth. The product of these probabilities is about 0.0004952. **(d)** Because there are four possible suits in which to have a flush, the probability of a flush is four times that found in part (c), or about 0.001981.

13.42 (Let D be the event a seedling was damaged by a deer.) **(a)** $P(D)$ = 209/871 = 0.2400. **(b)** The conditional probabilities are $P(D \mid \text{no cover})$ = 60/211 = 0.2844; $P(D \mid \text{cover} < 1/3)$ = 76/234 = 0.3248; $P(D \mid 1/3 \text{ to } 2/3 \text{ cover})$ = 44/221 = 0.1991; and $P(D \mid \text{cover} > 2/3)$ = 29/205 = 0.1415. **(c)** Cover and damage are not independent; $P(D)$ decreases noticeably when thorny cover is 1/3 or more.

13.43 This conditional probability is $P(\text{cover} > 2/3 \mid \text{not } D)$ = 176/(151 + 158 + 177 + 176) = 176/662 = 0.2659, or 26.59%.

13.44 First, note that having no thorny cover means there is less than 1/3 thorny cover. So, this conditional probability is $P(\text{cover} < 1/3 \mid D)$ = (60 + 76)/(60 + 76 + 44 + 29) = 136/209 = 0.6507, or 65.07%.

(The Venn diagram for Exercises 13.45 to 13.47 is provided.)



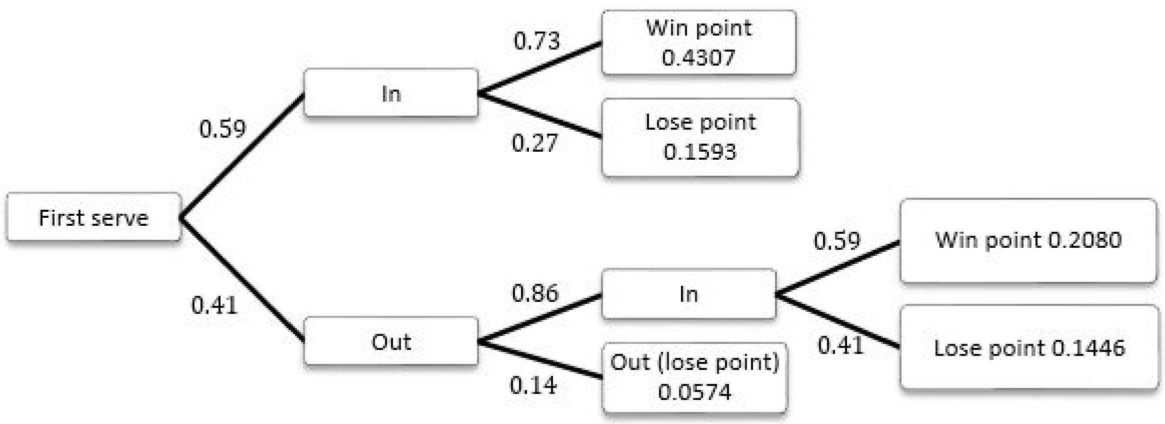
13.45 $P(\text{no products}) = 0.73$.

13.46 $P(B \text{ but not } A \text{ and not } C) = 0.1$.

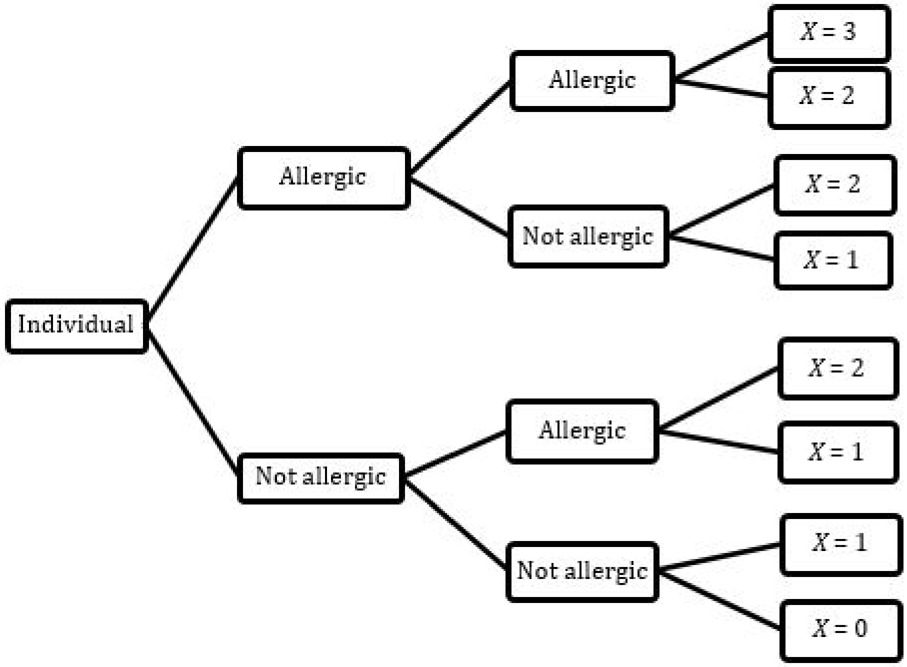
13.47 $P(A \mid B) = \frac{0.02 + 0.01}{0.1 + 0.02 + 0.01 + 0.03} = 0.1875$; $P(B \mid A) = \frac{0.02 + 0.01}{0.03 + 0.03 + 0.02 + 0.01} = 0.3333$.

13.48 **(a)** P(doubles on first toss) = 1/6, because 6 of the 36 equally likely outcomes enumerated in Figure 12.2 involve rolling doubles. **(b)** We need no doubles on the first roll (which happens with probability 5/6), then doubles on the second toss. P(first doubles appears on toss 2) = (5/6)(1/6) = 5/36. **(c)** Similarly, P(first doubles appears on toss 3) = $(5/6)^2(1/6) = 25/216$. **(d)** P(first doubles appears on toss 4) = $(5/6)^3(1/6)$, etc. In general, P(first doubles appears on toss k) = $(5/6)^{k-1}(1/6)$. **(e)** P(go again within 3 turns) = P(roll doubles in 3 or fewer rolls) = P(roll doubles on first, second, or third try) = (1/6) + (5/6)(1/6) + $(5/6)^2(1/6) = 0.4213$.

13.49 The tree diagram provided organizes this information; the probability of each outcome is the product of the individual branch probabilities leading to it. The total probability of the serving player winning a point is 0.4307 + 0.208034 = 0.638734.



13.50 PLAN: We construct a tree diagram showing the results (allergic or not) for each of the three individuals. SOLVE: In the tree diagram, each up-step represents an allergic individual (and has probability 0.02), and each down-step is a non-allergic individual (and has probability 0.98). At the end of each of the eight complete branches is the value of X. Any branch with two up-steps and one down-step has probability $0.02^2 \times 0.98^1 = 0.000392$ and yields $X = 2$. Any branch with one up-step and two down-steps has probability $0.02^1 \times 0.98^2 = 0.019208$ and yields $X = 1$. There are three branches each corresponding to $X = 2$ and $X = 1$, and only one branch each for $X = 3$ and $X = 0$. Because $X = 0$ and $X = 3$ appear on one branch each, $P(X = 0) = 0.98^3 = 0.941192$ and $P(X = 3) = 0.02^3 = 0.000008$. Meanwhile, $P(X = 1) = 3(0.02)^1(0.98)^2 = 0.057624$, and $P(X = 2) = 3(0.02)^2(0.98)^1 = 0.001176$. CONCLUDE: $P(X = 0) = 0.941192$, $P(X = 1) = 0.057624$, $P(X = 2) = 0.001176$, and $P(X = 3) = 0.000008$.



13.51 $P(\text{first serve in} \mid \text{server won point}) = \frac{P(\text{first serve in and server won point})}{P(\text{server won point})} = \frac{0.4307}{0.638734} =$ 0.6743, or 67.43%.

13.52 $P(X = 2|X \geq 1) = \frac{P(X=2 \text{ and } X \geq 1)}{P(X \geq 1)} = \frac{P(X=2)}{P(X \geq 1)} = \frac{0.001176}{1-0.941192} = 0.020.$

13.53 **[**For a randomly selected resident of the United States, let W, B, A, and L be (respectively) the events that this person is white, black, Asian, and lactose intolerant. We have been given:

$P(W) = 0.82$ $\quad P(B) = 0.14$ $\quad P(A) = 0.04$
$P(L|W) = 0.15$ $\quad P(L|B) = 0.70$ $\quad P(L|A) = 0.90$.]

(a) $P(L) = (0.82)(0.15) + (0.14)(0.70) + (0.04)(0.90) = 0.257$, or 25.7%.
(b) $P(A \mid L) = P(A \text{ and } L)/P(L) = (0.04)(0.90)/0.257 = 0.1401$, or 14%.

13.54 $P(D|A) = \frac{P(A|D)P(D)}{P(A|D)P(D)+P(A|I)P(I)+ P(A|R)P(R)+P(A|O)P(O)} = \frac{(0.23)(0.32)}{(0.23)(0.32)+(0.18)(0.45)+(0.04)(0.21)+(0.15)(0.02)} = 0.4434.$

13.55 **(a)** $P(A) = 0.26;\ P(B) = 0.49;\ P(M) = 0.2;\ P(D) = 0.05.$ **(b)** $P(A|F) = \frac{P(F|A)P(A)}{P(F|A)P(A)+P(F|B)P(B)+P(F|M)P(M)+P(F|D)P(D)} = \frac{(0.61)(0.26)}{(0.61)(0.26)+(0.57)(0.49)+(0.60)(0.20)+(0.52)(0.05)} =$ 0.2716;

$$P(B|F) = \frac{(0.57)(0.49)}{(0.61)(0.26) + (0.57)(0.49) + (0.60)(0.20) + (0.52)(0.05)} = 0.4783;$$

$$P(M|F) = \frac{(0.6)(0.2)}{(0.61)(0.26) + (0.57)(0.49) + (0.60)(0.20) + (0.52)(0.05)} = 0.2055;$$
$$P(D|F) = \frac{(0.52)(0.05)}{(0.61)(0.26) + (0.57)(0.49) + (0.60)(0.20) + (0.52)(0.05)} = 0.0445.$$
Explanations will vary.

13.56 In this problem, allele 29 is playing the role of A, and 0.181 is the proportion with this allele (a = 0.181). Similarly, allele 31 is playing the role of B, and the proportion having this allele is b = 0.071. The labels aren't important—you can reverse assignments of A and B. The proportion of the population with combination (29, 31) is therefore 2(0.181)(0.071) = 0.025702. The proportion with combination (29, 29) is (0.181)(0.181) = 0.032761. Of course, under these assignments, there are other alleles possible for this locus.

13.57 The proportion having combination (16, 17) is 2(0.232)(0.213) = 0.098832. (See Exercise 13.54 for the explanation of this calculation.)

13.58 In Exercise 13.56, we found that the proportion of the population with allele (29, 31) at loci D21S11 is 0.025702. In Exercise 13.57, we found that the proportion with allele (16, 17) at loci D3S1358 is 0.098832. Assuming independence between loci, the proportion with allele (29, 31) at D21S11 and (16, 17) at D3S1358 is (0.098832)(0.025702) = 0.002540.

13.59 If the DNA profile found on the hair is possessed by 1 in 1.6 million individuals, then we would expect about 3 individuals in the database of 4.5 million convicted felons to demonstrate a match. This comes from (4.5 million)/(1.6 million) = 2.8125, which was rounded up to 3.