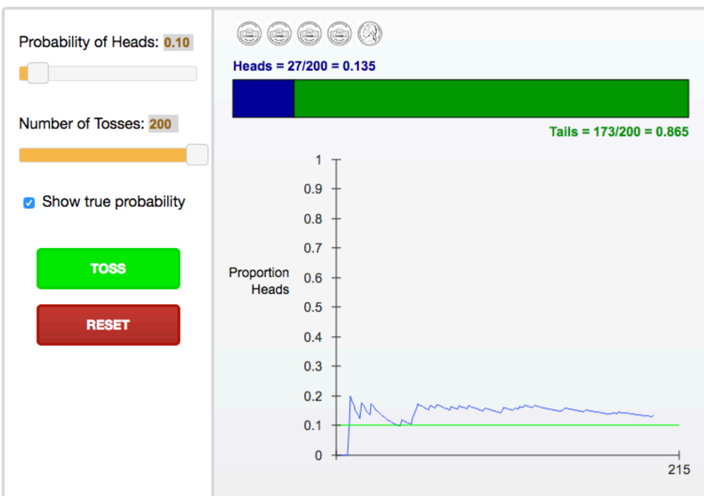
Chapter 12 – Introducing Probability

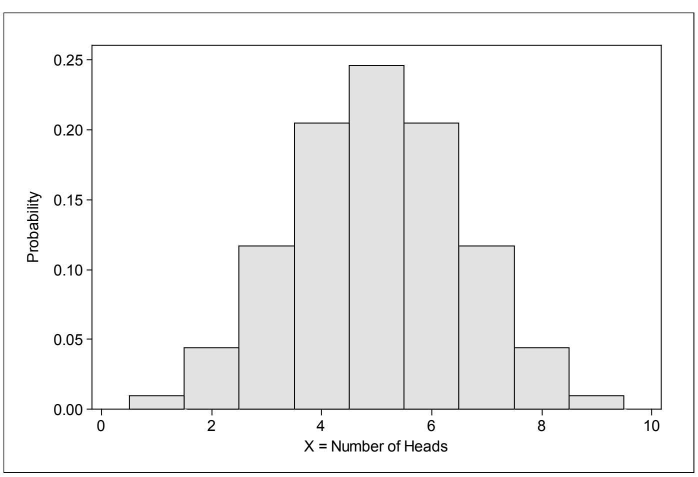
12.1 In the long run, with a *large* number of five-card poker hands, the fraction in which you will be dealt four-of-a-kind is 1/4165. It *does not* mean that exactly 1 out of 4165 such hands would yield four-of-a-kind. The probability of an event is the long-run frequency of times the event occurs if the experiment is repeated *endlessly* … not just 4000 times.

12.2 (a) An impossible event has probability 0. **(b)** A certain event has probability 1. **(c)** A probability of 0.99 would correspond to an event that is very likely but will not occur once in a while in a long sequence of trials. **(d)** An event with probability 0.45 will occur slightly less often than it does not occur; it will occur a bit less than 50% of the time.

12.3 (a) There are 21 zeros among the first 200 digits of the table (rows 101–105), for a proportion of 0.105. **(b)** Answers will vary, but more than 99% of all students should get between 7 and 33 heads out of 200 flips when $p = 0.1$.



12.4 (a) In about 95% of all simulations, there will be between 2 and 8 heads, so the sample proportion will be between 0.2 and 0.8. **(b)** Shown is the theoretical histogram; a stemplot of 25 proportions will have roughly that shape.
12.5 (a) S = {owns a car, does not own a car}. **(b)** S = {All numbers between ______ and



_______ cm}. (Choices of upper and lower limits will vary.) **(c)** *S* = {000, 001, 002, …, 999}. **(d)** *S* = {January, February, …, December}

12.6 (a) The accompanying table illustrates the 16 possible pair combinations in the sample space.

	1	2	3	4
1	1 1	1 2	1 3	1 4
2	2 1	2 2	2 3	2 4
3	3 1	3 2	3 3	3 4
4	4 1	4 2	4 3	4 4

(b) Each of the 16 outcomes has probability 1/16.

12.7 For the sample space, add 2 to each pair total in the table shown in the previous solution: *S* = {4, 5, 6, 7, 8, 9, 10}. As all faces are equally likely and the dice are independent, each of the 16 possible pairings is equally likely, so (for example) the probability of a total of 6 is 3/16, because 3 pairings add to 4 (and then we add 2). The complete set of probabilities is shown in the table.

Total	Probability
4	1/16 = 0.0625
5	2/16 = 0.1250
6	3/16 = 0.1875
7	4/16 = 0.2500
8	3/16 = 0.1875
9	2/16 = 0.1250
10	1/16 = 0.0625

12.8 (a) 2.5% + 34.2% + 3.2% = 39.9%. This makes use of Rule 3, because being from Canada, for example, means you are not from Mexico or the Caribbean. A student cannot be from more than one country. **(b)** 65.8% (100% − 34.2%) were not from the United States. This makes use of Rule 4.

12.9 (a) Event *B* specifically rules out obese subjects, so there is no overlap with event *A*. **(b)** *A* or *B* is the event "The person chosen is overweight or obese." *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) = 0.38 + 0.33 = 0.71. **(c)** *P*(*C*) = 1 − *P*(*A* or *B*) = 1 − 0.71 = 0.29.

12.10 (a) *P*(either English or French) = 0.578 + 0.217 = 0.795. **(b)** *P*(other language) = 1 − 0.795 = 0.205. **(c)** *P*(not English) = 1 − 0.578 = 0.422. (Or, add the other two probabilities.)

12.11 (a) Disjoint. **(b)** Not disjoint; $300,000 is more than $100,000 and more than $250,000. **(c)** Disjoint; 3 + *x* cannot equal 3.

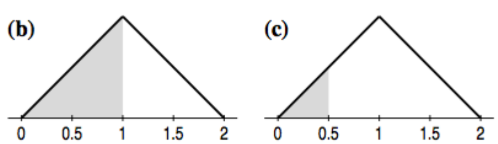
12.12 Model 1: not valid (probabilities have the sum 6/7). Model 2: valid. Model 3: not valid (probabilities have the sum 7/6). Model 4: not valid (probabilities cannot be more than 1).

12.13 (a) A = {4, 5, 6, 7, 8, 9}, so $P(A)$ = 0.097 + 0.079 + 0.067 + 0.058 + 0.051 + 0.046 = 0.398. **(b)** B = {2, 4, 6, 8}, so $P(B)$ = 0.176 + 0.097 + 0.067 + 0.051 = 0.391. **(c)** A or B = {2, 4, 5, 6, 7, 8, 9}, so $P(A$ or $B)$ = 0.176 + 0.097 + 0.079 + 0.067 + 0.058 + 0.051 + 0.046 = 0.574. This is different from $P(A) + P(B)$ because A and B are not disjoint.

12.14 (a) This is a valid finite probability model, because the probabilities are all greater than 0 and sum to 1. **(b)** The event $\{X < 4\}$ is the event in which an individual drinks fewer than 4 cups of coffee on average per day. $P(X < 4)$ = 0.36 + 0.26 + 0.19 + 0.08 = 0.89. Alternatively, $P(X < 4) = 1 - P(X \geq 4) = 1 - 0.11 = 0.89$. **(c)** This is the event $\{X \geq 1\}$. $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.36 = 0.64$.

12.15 (a) $P(Y \leq 0.6) = 0.6$. **(b)** $P(Y < 0.6) = 0.6$. The only difference between parts (a) and (b) is the inclusion of the point $Y = 0.6$. This has 0 probability for a continuous variable. **(c)** $P(0.4 \leq Y \leq 0.8) = 0.4$. **(d)** $P(0.4 < Y \leq 0.8) = 0.4$. The only difference between parts (c) and (d) is the inclusion of the point $Y = 0.4$. This has 0 probability for a continuous variable.

12.16 (a) The area of a triangle is $\frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$. **(b)** $P(X < 1) = 0.5$. **(c)** $P(X < 0.5) = 0.125$.



12.17 (a) $\{X \geq 510\}$ **(b)** $P(X \geq 510) = P(Z \geq \frac{510 - 499.6}{10.4}) = P(Z \geq 1) = 1 - 0.8413 = 0.1587$ (using Table A).

12.18 (a) X is a finite random variable, because the sample space is finite. **(b)** $P(X \geq 3.0)$ is "the probability that a randomly selected Economics 201 student earned B or better (that is, B, B+, A−, A, or A+)." $P(X \geq 3)$ = 0.16 + 0.09 + 0.09 + 0.06 + 0.02 = 0.42. **(c)** $\{X < 2.7\}$. We want $P(X < 2.7) = P(X \leq 2.3)$ = 0.05 + 0.04 + 0.05 + 0.03 + 0.07 + 0.09 + 0.11 = 0.44.

12.19 (a) Y is a continuous random variable, because it can take on any value within an interval. **(b)** $P(Y \geq 8)$ is "the probability that a randomly selected able-bodied male student runs a mile in 8 minutes or more." $P(Y \geq 8) = P(Z \geq \frac{8-7.11}{0.74}) = P(Z \geq 1.20) = 1 - 0.8849 = 0.1151$ (using Table A). **(c)** $\{Y < 6\}$. $P(Y < 6) = P\left(Z < \frac{6-7.11}{0.74}\right) = P(Z < -1.50) = 0.0668$ (using Table A).

12.20 (a) Answers will vary (probably wildly). **(b)** A personal probability might take into account specific information about one's own driving habits or about the kind of traffic in which one usually drives. **(c)** Most people believe that they are better-than-average drivers (whether or not they have any evidence to support that belief).

12.21 (a) If Joe says that *P*(Notre Dame wins) = 0.05, then he believes that *P*(North Carolina wins) = 0.1 and *P*(Duke wins) = 0.2. **(b)** Joe's probabilities for Duke, Notre Dame, and North Carolina add up to 0.35, so that leaves probability 0.65 for all other teams.

12.22 (a) if you deal millions of poker hands, the fraction of them that contain a straight flush will be very close to 1/64,974. Probabilities express the *approximate* fraction of occurrences out of many trials.

12.23 (b) *S* = whole numbers 0 to 6. The set {0, 1, 2, 3, 4, 5, 6} lists all possible counts.

12.24 (b) finite. This is a finite model with a limited number of outcomes.

12.25 (b) 0.02. The other probabilities add to 0.98, so this must be 0.02.

12.26 (b) 0.59. *P*(Republican or Democrat) = *P*(Republican) + *P*(Democrat) = 0.28 + 0.31 = 0.59.

12.27 (b) 0.72. *P*(not Republican) = 1 − *P*(Republican) = 1 − 0.28 = 0.72.

12.28 (b) 1/10. There are 10 equally likely possibilities, so *P*(digit in table is 7) = 1/10.

12.29 (c) 3/10. "7 or greater" means 7, 8, or 9 (3 of the 10 possibilities).

12.30 (b) 22.7%. 1 – (0.087 + 0.323 + 0.363) = 0.227, or 22.7%, have three or more cars.

12.31 (c) 0.005. $Y > 1$ standardizes to $Z > 2.56$, for which Table A gives 0.0052.

12.32 (a) There are 16 possible outcomes: {HHHH, HHHM, HHMH, HMHH, MHHH, HHMM, HMHM, HMMH, MHHM, MHMH, MMHH, HMMM, MHMM, MMHM, MMMH, MMMM}. **(b)** The sample space is {0, 1, 2, 3, 4}.

12.33 (a) Legitimate (even though it is not a "fair" die). **(b)** Legitimate (even if the deck of cards is not!). **(c)** Not legitimate (the total is more than 1).

12.34 (a) The given probabilities have the sum 0.71, so this probability must be 1 – 0.71 = 0.29. **(b)** *P*(at least a high school education) = 1 − *P* (has not finished high school) = 1 − 0.10 = 0.90. (Or, add the other three probabilities.)

12.35 (In computing the probabilities, we have dropped the trailing zeros from the land area figures.) **(a)** *P*(area is forested) = 4176/9094 = 0.4592. **(b)** *P*(area is not forested) = 1 − 0.4592 = 0.5408.

12.36 (a) All probabilities are between 0 and 1, and they add to 1. (We must assume that no student takes more than one language.) **(b)** The probability that a student is studying a language other than English is 0.43 = 1 − 0.57. Or, add all the other probabilities. **(c)** This probability is 0.40 = 0.30 + 0.08 + 0.02.

12.37 (a) The given probabilities add to 0.98, so other colors must account for the remaining 0.02. **(b)** Because *P*(white or silver) = 0.35 + 0.12 = 0.47, the complement rule gives *P*(neither white nor silver) = 1 − 0.47 = 0.53.

12.38 (Of the seven cards, there are three 9s, two red 9s, and two 7s.) **(a)** *P*(draw a 9) = 3/7. **(b)** *P*(draw a red 9) = 2/7. **(c)** *P*(don't draw a 7) = 1 − *P*(draw a 7) = 1 − 2/7 = 5/7.

12.39 The probabilities of 2, 3, 4, and 5 are unchanged (1/6), so *P*(1 or 6) must still be 1/3. If *P*(6) = 0.2, then *P*(1) = 1/3 − 0.2 = 2/15.

Face	⚀	⚁	⚂	⚃	⚄	⚅
Probability	$0.1\overline{3}$	1/6	1/6	1/6	1/6	0.2

12.40 Each of the 90 guests has probability 1/90 of winning the prize. The probability that the winner is a woman is the sum of 1/90 42 times, once for each woman. The probability is 42/90 = 0.467.

12.41 (a) It is legitimate, because every person must fall into exactly one category, the probabilities are all between 0 and 1, and they add up to 1. **(b)** 0.176 = 0.002 + 0.008 + 0.155 + 0.011 is the probability that a randomly chosen American is Hispanic. **(c)** 0.384 = 1 − 0.616 is the probability that a randomly chosen American is *not* a non-Hispanic white.

12.42 (a) It is legitimate, because every person must fall into exactly one category, the probabilities are all between 0 and 1, and they add up to 1. **(b)** *P*(20- to 24-year-old who is married) = 0.027 **(c)** *P*(20- to 24-year-old) = 0.260, which is the sum of the numbers in the first column. **(d)** *P*(married) = 0.398, which is the sum of the numbers in the second row.

12.43 (a) *A* corresponds to the outcomes in the first column and the second row. **(b)** Adding up those six outcomes gives *P*(*A*) = 0.631. This is different from the sum of the probabilities in parts (c) and (d) of Exercise 12.42 because that sum counts the overlap (0.027) twice.

12.44 (a) *P*(30 years old or older) = 0.483 (the sum of the entries in the third and fourth columns). **(b)** *P*(is or has been married) = 0.474 (the sum of the entries in the second and third rows).

12.45 (a) X is discrete, because it has a finite sample space. **(b)** "At least one nonword error" is the event $\{X \geq 1\}$ (or $\{X > 0\}$). $P(X \geq 1) = 1 - P(X = 0) = 0.9$. **(c)** $\{X \leq 2\}$ is "no more than two nonword errors," or "fewer than three nonword errors." $P(X \leq 2) = P(X = 0) + P(X$

= 1) + $P(X = 2) = 0.1 + 0.2 + 0.3 = 0.6$. $P(X < 2) = P(X = 0) + P(X = 1) = 0.1 + 0.2 = 0.3$.

12.46 (a) All nine digits are equally likely, so each has probability 1/9:

Value of W	1	2	3	4	5	6	7	8	9
Probability	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

(b) $P(W \geq 6) = 4/9 = 0.444$, or twice as big as the Benford's law of probability.

12.47 (a) There are ten pairs. Just using initials: {(A, D), (A, M), (A, S), (A, R), (D, M), (D, S), (D, R), (M, S), (M, R), (S, R)}. **(b)** Each has probability 1/10 = 10%. **(c)** Mei-Ling is chosen in four of the ten possible outcomes: 4/10 = 40%. **(d)** There are three pairs with neither Sam nor Roberto, so the probability is 3/10.

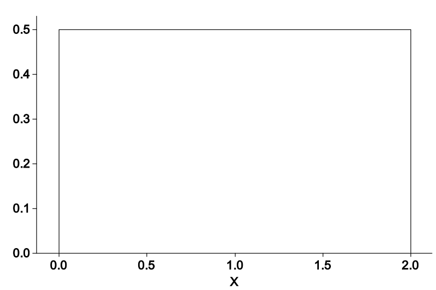
12.48 (a) BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG. Each has probability 1/8. **(b)** Three of the eight arrangements have two (and only two) girls, so $P(X = 2) = 3/8 = 0.375$. **(c)** See table.

Value of X	0	1	2	3
Probability	1/8	3/8	3/8	1/8

12.49 The possible values of Y are 1, 2, 3, . . . , 12, each with probability 1/12. Aside from drawing a diagram showing all the possible combinations, one can reason that the first (regular) die is equally likely to show any number from 1 through 6. Half of the time, the second roll shows 0, and the other half it shows 6. Each possible outcome, therefore, has probability (1/6)(1/2) = 1/12.

12.50 (a) There are 20 ways to choose three of the six cups. (See Example 12.8.) **(b)** If we consider the three cups with the milk added first as only one of the 20 possible combinations of milk first (or last), this probability is 1/20.

12.51 (a) This is a continuous random variable, because the set of possible values is an interval. **(b)** The height should be 1/2 because the area under the curve must be 1. (For a rectangle, area = L × W.) The density curve is illustrated.



(c) $P(Y \leq 1) = 1/2$.

12.52 (For these probabilities, compute the areas of the appropriate rectangle under the density shown for Exercise 12.51.) **(a)** $P(0.5 < Y < 1.3) = (0.8)(0.5) = 0.4$. **(b)** $P(Y \geq 0.8) = (1.2)(0.5) = 0.6$.

12.53 (a) $P(0.49 \leq V \leq 0.53) = P\ (\frac{0.49-0.51}{0.009} \leq Z \leq \frac{0.53-0.51}{0.009}) = P(-2.22 \leq Z \leq 2.22) = 0.9868 - 0.0132 = 0.9736$. **(b)** $P(V \leq 0.49) = P\ (Z \leq \frac{0.49-0.51}{0.009}) = P(Z \leq -2.22) = 0.0132$.

12.54 $P(8.9 \leq X \leq 9.1) = P\ (\frac{8.9-9}{0.075} \leq Z \leq \frac{9.1-9}{0.075}) = P(-1.33 \leq Z \leq 1.33) = 0.9082 - 0.0918 = 0.8164$.

12.55 (a) Because there are 10,000 equally likely four-digit numbers (0000 through 9999), the probability of an exact match is 1/10,000 = 0.0001. **(b)** There is a total of 24 = 4 × 3 × 2 × 1 arrangements of the four digits 5, 9, 7, and 4 (there are four choices for the first digit, three for the second, and two for the third), so the probability of a match in any order is 24/10,000 = 0.0024.

12.56 Note that, in this experiment, factors other than the nickel's characteristics might affect the outcome. For example, if the surface used is not quite level, there will be a tendency for the nickel to fall in the "downhill" direction.

12.57 (a) to **(c)**: Results will vary, but after n tosses, the distribution of the proportion (call it $\hat{p}$) is approximately Normal, with mean 0.5 and standard deviation $1/(2\sqrt{n})$, while the distribution of the count of heads is approximately Normal, with mean $0.5n$ and standard deviation $\sqrt{n}/2$. Therefore, using the 68–95–99.7 rule, we have the results shown in the table. Note that the range for the proportion $\hat{p}$ gets narrower, while the range for the count gets wider.

n	99.7% range for $\hat{p}$	99.7% range for count
40	0.5 ± 0.237	20 ± 9.5
120	0.5 ± 0.137	60 ± 16.4
240	0.5 ± 0.097	120 ± 23.2
480	0.5 ± 0.068	240 ± 32.9

12.58 (a) Virtually all answers will be between 62% and 88%, and about 95% of the students' answers will be between 66% and 84%. **(b)** Answers will vary, of course. Many students should have longest runs of "made" shots longer than 6.

12.59 (a) With $n = 50$, the variability in the proportion (call it $\hat{p}$) is larger. With $n = 100$, nearly all answers will be between 0.24 and 0.52. With $n = 400$, nearly all answers will be between 0.31 and 0.45. **(b)** Results will vary.

12.60 is a Web-based exercise.